# DC NETWORK THEOREM

**Current:** "Rate of flow of electric charge."

$$I = \frac{\text{Charge}}{\text{time}} \qquad \text{Coulombs/Sec or Ampere}$$

**Note:-**

1. Direction of current is same as the direction of motion of +Ve charge or opposite to the direction of motion of –Ve charge.

**Voltage:** **"**Energy required in transferring a charge of one coulomb from one point to another point."

$$V = \frac{\text{Energy(W)}}{\text{Charge(Q)}} \qquad \text{Joule/Coulomb or Volts}$$

**EMF (Electromotive force):** "The EMF of a voltage source is the energy imparted by the source to each coulomb of the charge passing through it."

$$E = \frac{\text{Energy(W)}}{\text{Charge(Q)}} \qquad \text{Joule/Coulomb or Volts}$$

**Potential Difference:** "The pd between two points is the energy required in transferring a charge of coulomb from one point to another point."

$$pd = \frac{\text{Energy(W)}}{\text{Charge(Q)}} \qquad \text{Joule/Coulomb or Volts}$$

**Voltage drop:** "The voltage drop between two points is the decrease in energy required in transferring a charge of coulomb from one point to another point."

$$\text{Voltage drop} = \frac{\text{Energy(W)}}{\text{Charge(Q)}} \qquad \text{Joule/Coulomb or Volts}$$

**Resistance:** "Electric resistance is the property of material which offers opposition to the flow of current and dissipates energy."

$$R = \rho\frac{l}{a} \qquad \text{Ohm or } \Omega \qquad \text{(Law of resistance)}$$

Where $l = \text{Length of the wire}$

$a = \text{cross} - \text{sectional area of the wire}$

$\rho = \text{Resistivity or Specific Resistance of the material}$

**Note:-**

1. Resistance also depends on temperature.

**Ohm's Law:** "The current passing through a conductor is directly proportional to potential drop across its ends provided physical conditions are same."

$$I \propto V$$

Or $$I = \frac{1}{R}V$$

Or $$V = IR$$

**Conductance (G):** It is reciprocal of resistance

$$G = \frac{1}{\text{Resistance (R)}} = \frac{a}{\rho l} = \sigma \frac{a}{l} \qquad \text{mho, } \Omega^{-1}\text{, Siemens}$$

**Power & Energy in an electric circuit:**

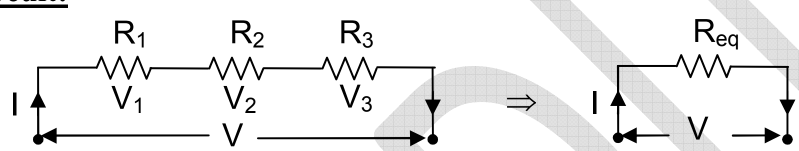
Power $P = VI$    Watt, KW, MW

$$= I^2 R$$

$$= \frac{V^2}{R}$$

Energy $E = P \times time$    Joule, watt-sec, watt-hours, KWH

**Series Circuit:**



(a) (b)

Fig.1

We know   $V_1 = IR_1$   $V_2 = IR_2$   $V_3 = IR_3$

Total voltage   $V = V_1 + V_2 + V_3$

$$\Rightarrow IR_{eq} = IR_1 + IR_2 + IR_3$$

$$\Rightarrow R_{eq} = R_1 + R_2 + R_3$$

**Note:-**

1. Same current means resistances are in series.

**Example 1:** Three resistors are connected in series across a 12V battery. The one resistance has a value of 1 ohm, second has a voltage drop of 4 Volts & third has power dissipation of 12 W. Calculate value of each resistance & circuit current.

**Solution:** Hint   $V = V_1 + V_2 + V_3$

$$12 = I + 4 + 12/I$$

I = 2 Or 6 Amperes

When I=2 Amp

$R_1 = 1$   $R_2 = 2$   $R_3 = 3$

When I=6 Amp

$R_1 = 1$   $R_2 = 2/3$   $R_3 = 1/3$

## Parallel Circuit:

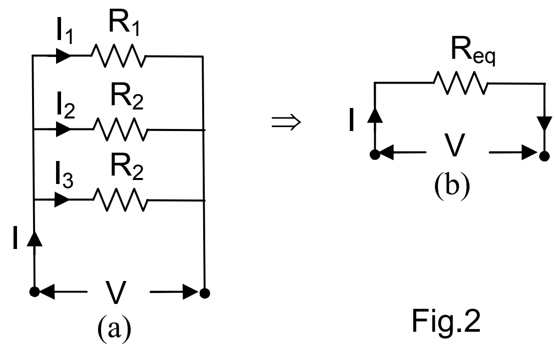



Fig.2

We know $I_1 = \dfrac{V}{R_1}$ $\quad I_2 = \dfrac{V}{R_2}$ $\quad I_3 = \dfrac{V}{R_3}$

Total current $\quad I = I_1 + I_2 + I_3$

$$\Rightarrow \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Note:-

1. Same voltage means resistances are in parallel.
2. If two resistances are in parallel and $R_1=R_2=R$ then

$$R_{eq} = \frac{R}{2}$$

## Series Parallel circuit:

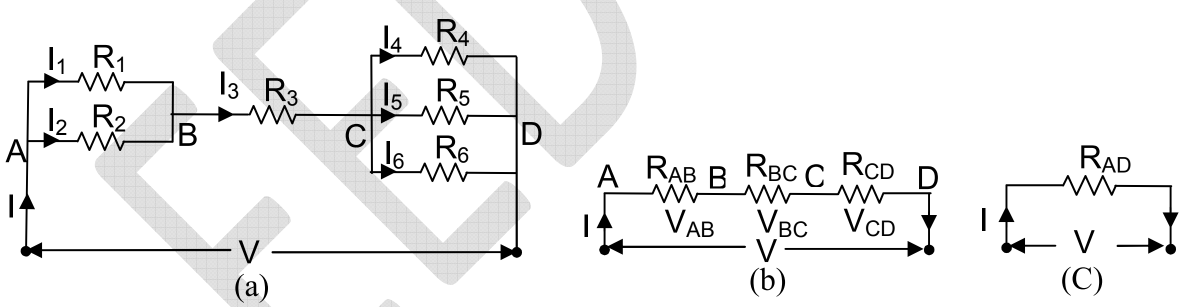



Fig.3

$R_{AB} = \dfrac{R_1 R_2}{R_1 + R_2}$ $\qquad R_{BC} = R_3$ $\qquad \dfrac{1}{R_{CD}} = \dfrac{1}{R_4} + \dfrac{1}{R_5} + \dfrac{1}{R_6}$

So $\quad R_{AD} = R_{AB} + R_{BC} + R_{CD}$ And $\quad I = \dfrac{V}{R_{AD}}$

$V_{AB} = IR_{AB}$ From fig.3b

$V_{BC} = IR_{BC}$ From fig.3b

$V_{CD} = IR_{CD}$ From fig.3b

$$I_1 = \frac{V_{AB}}{R_1} \qquad I_2 = \frac{V_{AB}}{R_2} \qquad I_3 = \frac{V_{BC}}{R_3} = I \qquad \text{From fig.3a}$$

$$I_4 = \frac{V_{CD}}{R_4} \qquad I_5 = \frac{V_{CD}}{R_5} \qquad I_6 = \frac{V_{CD}}{R_6} \qquad \text{From fig.3a}$$

Power consumed by whole circuit

$$P = I^2 R_{AD} \qquad \text{From fig.3c}$$

Or $$= I^2 R_{AB} + I^2 R_{BC} + I^2 R_{CD} \qquad \text{From fig.3b}$$

Or $$= I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 + I_5^2 R_5 + I_6^2 R_6 \qquad \text{From fig.3a}$$

**Example 2:** Two resistances of 20 & 30 ohms respectively are connected in parallel. These two parallel resistances are further connected in series with a third resistance of 15 ohm. If current through 15 ohm resistance is 3 Amperes. Find

(i) Current through 20 & 30 ohm resistances
(ii) Voltage across whole circuit
(iii) Total power consumed.

**Solution:** Hint

$R_{eq} = (20\|30) + 15$
$= 12 + 15 = 37$

$V_{AB} = 12 \text{x} 3 = 36$

(i) $I_1 = 36/20 = 1.8\,A \qquad I_2 = 36/30 = 1.2\,A$

(ii) $V_{AC} = V_{AB} + V_{BC} = 36 + 15 \times 3 = 81 Volts$

(iii) Power = VI = 243 Watts

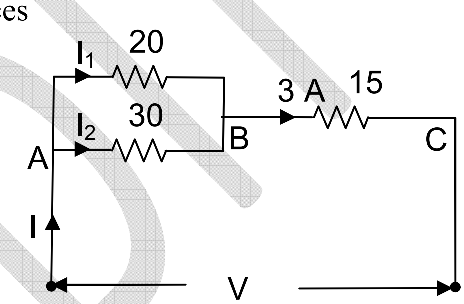


Fig. 4

**<u>Circuit or Network Elements:</u>** R, L, C

**<u>Network or Circuit:</u>** "Path followed by an electric current."

Or

"Any interconnection of circuit elements with or without energy sources."

**Note:-**

1. A circuit must have a closed energized path.
2. A network may not have a closed path i.e. T-Network

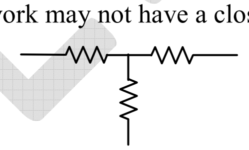


Fig.5-T-Network

3. So every network may not be circuit (i.e. T-Network) but every circuit is a network.

**<u>Loop:</u>** "Any closed path in the network."

**<u>Mesh:</u>** "A closed path which does not have any closed path inside it."

**<u>Node:</u>** "It is a junction where 2 or more branches are connected together."

**Note:-**

1. A junction where 3 or more branches connected together is known as principal node or essential node.
2. For example

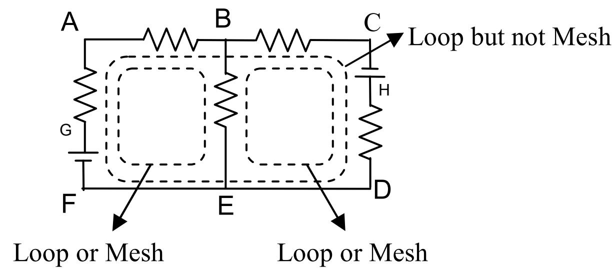



Fig.6

| | | |
|---|---|---|
| Total no of nodes | = 6+2 = 8 | (A, B, C, D, E, F, G, H) |
| No of principal nodes | = 2 | (B & E) |
| No of Loops | = 3 | (ABEFA, BCDEB, ABCDEFA) |
| No of Meshes | = 2 | (ABEFA, BCDEB) |

**Energy Sources:**

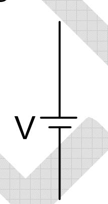
Basically Two types

(A) Independent Energy Sources
- a. Voltage Source
  - i. AC/DC
  - ii. Ideal/Practical
- b. Current Source
  - i. AC/DC
  - ii. Ideal/Practical

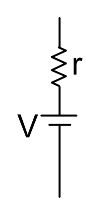
(B) Dependent Energy Sources
- a. Voltage Source
- b. Current Source

(A) Independent Energy Sources:

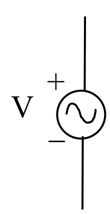
a. Voltage Source




(a) Ideal DC Voltage Source
(Internal Resistance, r=0)

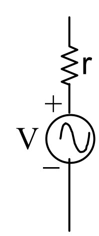



(b) Practical DC Voltage Source
(With Internal Resistance, r)



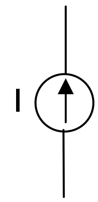

(c)Ideal AC Voltage Source
(Internal Resistance, r=0)

Fig.7

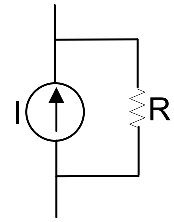



(d) Ideal AC Voltage Source
(Internal Resistance, r)

b. Current Source



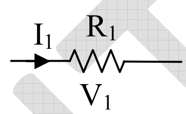

(a) Ideal Current Source
(Internal Resistance, R=infinity)

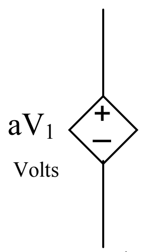



(b) Practical Current Source
(Internal Resistance, R)

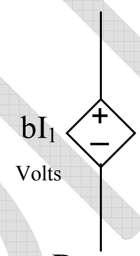
Fig.8

(B) Dependent Energy Sources
Consider some branch of a circuit

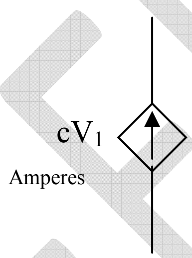


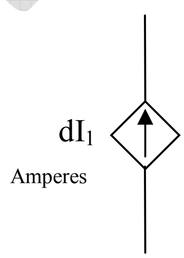

(a) Voltage Dependent Voltage Source (VDVS)




(b) Current Dependent Voltage Source (CDVS)




(c) Voltage Dependent Current Source (VDCS)




(d) Current Dependent Current Source (CDCS)

Fig.9

**Kirchhoff's Current Law (KCL):**
"At any instant of time, the algebraic sum of currents at any node is zero."

$$\sum I = 0$$

Or

"Total incoming currents = Total outgoing currents."

**Kirchhoff's Voltage Law (KVL):**
"At any instant of time, the algebraic sum of voltages in a closed path is zero."

Consider following circuit

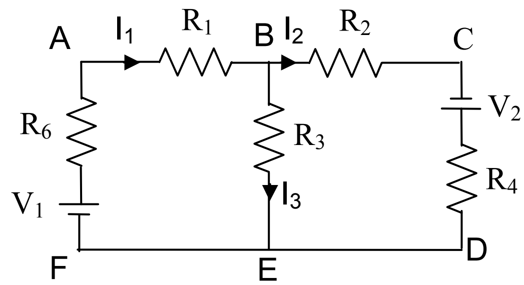



Fig.10

Apply KCL at node B $I_1 - I_2 - I_3 = 0$ Or $I_1 = I_2 + I_3$

Apply KVL in loop ABEFA

$$I_1R_1 + I_3R_3 - V_1 + I_1R_6 = 0$$

Apply KVL in loop BCDEB

$$I_2R_2 - V_2 + I_2R_4 - I_3R_3 = 0$$

Apply KVL in loop ABCDEFA

$$I_1R_1 + I_2R_2 - V_2 + I_2R_4 - V_1 + I_2R_6 = 0$$

**Example 3:** In the following circuit calculate current in each branch by KCL & KVL (Fig.11a).

**Solution:**

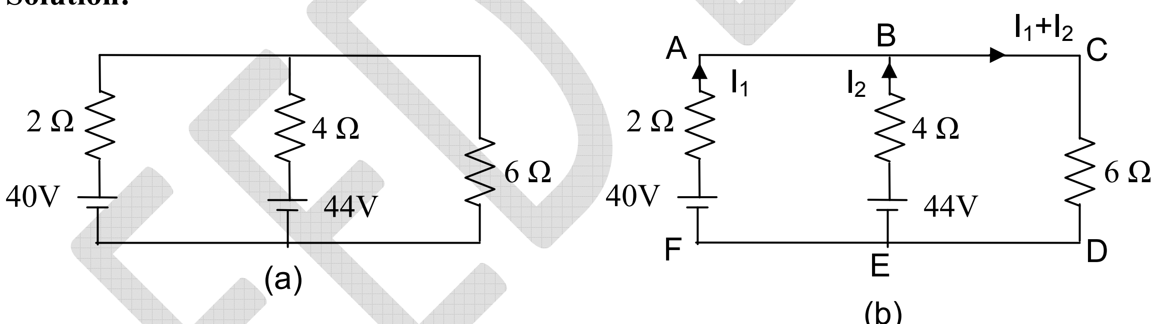



Fig.11

Apply KVL in loop ABEFA $-40 + 2I_1 - 4I_2 + 44 = 0$ --------(1)

Apply KVL in loop BCDEB $-44 + 4I_2 + 6(I_1 + I_2) = 0$ -------(2)

Solving (1) & (2) $I_1 = \frac{34}{11}A$ $I_2 = \frac{28}{11}A$ $I_1 + I_2 = \frac{62}{11}A$

**Mesh Analysis:**

Steps are

1. Identify the total meshes
2. Assume some mesh current in each mesh (clockwise or anticlockwise)
3. Apply KVL in each mesh.
4. Solve the above equations.

**Example 4:** Solve Example 3 by Mesh Analysis.
**Solution:**
Let the two meshes are having clockwise currents as shown in following figure

Apply KVL in loop 1

$$-40+2I_1-4(I_1-I_2)+44=0----(1)$$

Apply KVL in loop 2

$$-44+4(I_2-I_1)+6I_2=0----(2)$$

Solving (1) & (2)

$$I_1=\frac{34}{11}A \qquad I_2-I_1=\frac{28}{11}A \qquad I_2=\frac{62}{11}A$$

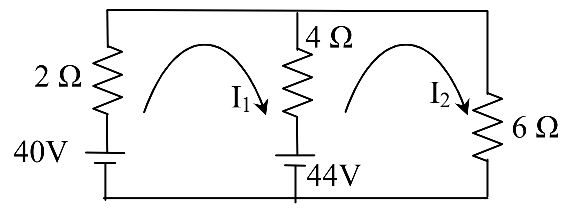
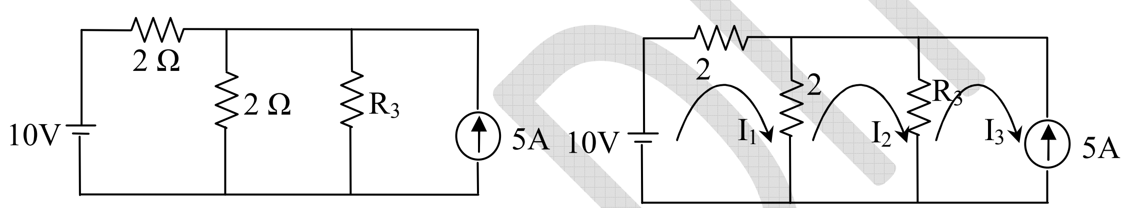


Fig.12

**Example 5:** Find the current through $R_3$=4 ohm resistance by mesh analysis (Fig.13a)
**Solution:**



(a) Fig.13 (b)

Let the three meshes are having clockwise currents as shown in following figure
Apply KVL in mesh 1

$$-10+2I_1+2(I_1-I_2)=0------(1)$$

Apply KVL in mesh 2

$$+2(I_2-I_1)+4(I_2-I_3)=0------(2)$$

Clearly From mesh 3

$$I_3=-5A------(3)$$

Solving (1), (2) & (3) $\qquad I_1=-1A \qquad I_2=-3$

Current in $R_3$ = $I_2$-$I_3$ = (-3)-(-5) = 2 A

**<u>Node Analysis:</u>**
Steps are

1. Identify the total **Principal Nodes**
2. Assume one node as reference node (Voltage of this node = 0 Volts)
3. Assume some node voltages for other remaining nodes w.r.to reference node. ($V_1$, $V_2$, $V_3$ etc).
4. Assume some branch currents in different branches.
5. Apply KCL at different nodes and make the equations in terms of node voltages and circuit elements.
6. Solve the above equations.

**Example 6:** Solve Example 3 by Node Analysis.
**Solution:**

Apply KCL at node $V_1$

$$\left(\frac{V_1-40-0}{2}\right)+\left(\frac{V_1-44-0}{4}\right)+\left(\frac{V_1-0}{6}\right)=0$$

$$V_1=\frac{372}{11}Volts$$

So $$I_1=-\frac{34}{11}A$$

$$I_2=-\frac{28}{11}A$$

$$I_3=\frac{62}{11}A$$

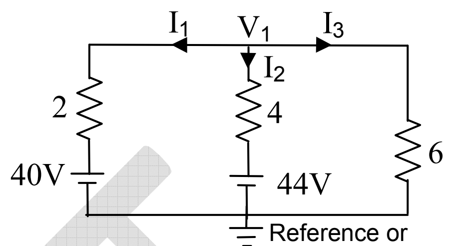



Fig.14

**Example 7:** Solve Example 5 by Node Analysis.
**Solution:**

Apply KCL at node $V_1$

$$\left(\frac{V_1-10-0}{2}\right)+\left(\frac{V_1-0}{2}\right)+\left(\frac{V_1-0}{4}\right)=5$$

$$V_1=8\,Volts$$

So $$I_3=\left(\frac{V_1-0}{4}\right)=2\ Amps$$

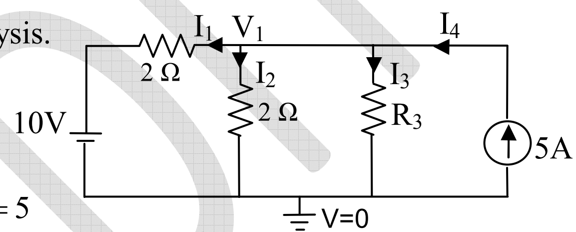
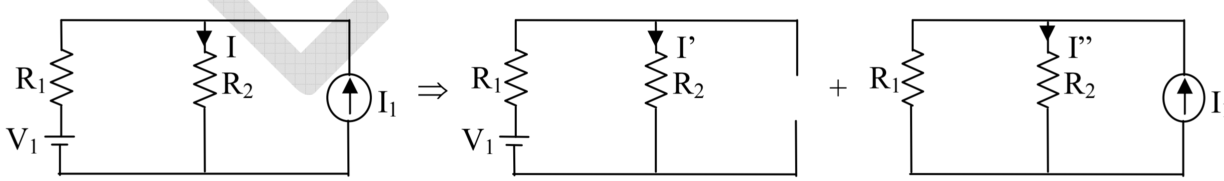



Fig.15

**<u>Superposition theorem:</u>**

"In a linear circuit, containing more then one independent energy sources, the overall response (Voltage or current) in any branch of the circuit is equal to sum of the response due to each independent source acting one at a time while making other source in-operative."




(a) (b) (c)

Fig.16

So according to superposition theorem $I=I'+I''$

**Example 8:** Solve example 5 by superposition theorem.
**Solution:**

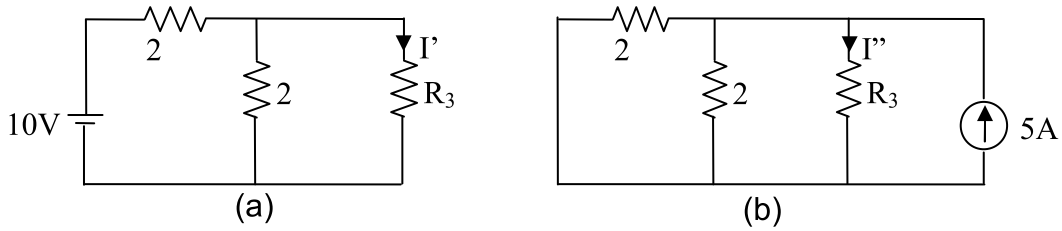



Fig.17

<u>Consider 10 V Voltage source only (Fig.17a)</u>

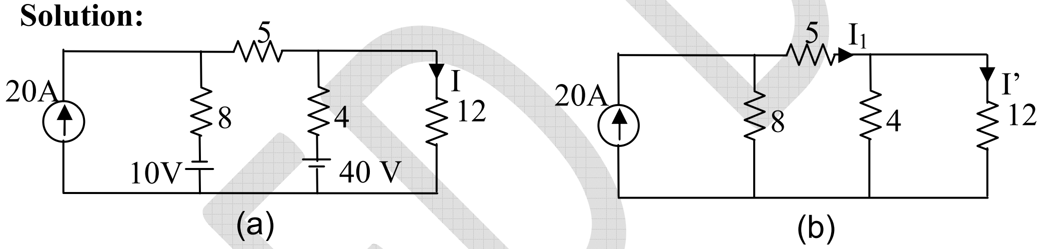
$$I' = \left(\frac{2}{2+4}\right)\left(\frac{10}{2+(2\,||\,4)}\right) = 1\,Amps$$

<u>Consider 5A Current Source only (Fig.17b)</u>

$$I'' = \left(\frac{1}{1+4}\right)5 = 1\,Amps$$

So I=I'+I''=1+1=2 Amps

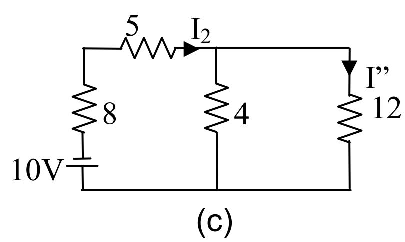
**Example 9:** Find I in the following circuit by superposition theorem (Fig.18a).
**Solution:**




<u>*Consider 20A current source only* (Fig.18b)</u>

By current division rule

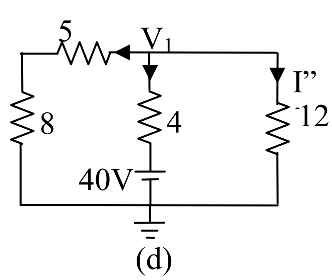
$$I_1 = \left(\frac{8}{8+5+(4\,||\,12)}\right)20 = 10$$




Again bye current division rule

$$I' = \left(\frac{4}{4+12}\right)10 = \frac{5}{2}\,Amps$$

<u>*Consider 10V Voltage source only* (Fig.18c)</u>




$$I_2 = -\left(\frac{10}{8+5+(4\,||\,12)}\right) = -\frac{5}{8}$$

By current division rule

Fig.18

$$I'' = \left(\frac{4}{4+12}\right)\left(-\frac{5}{8}\right) = -\frac{5}{32}\,Amps$$

*Consider 40V Voltage source only* (Fig.18d)

Apply KCL at node $V_1$

$$\left(\frac{V_1 - 0}{8+5}\right) + \left(\frac{V_1 - 40 - 0}{4}\right) = \left(\frac{V_1 - 0}{12}\right)$$

$$V_1 = \frac{6240}{256} V$$

$$I''' = \left(\frac{V_1 - 0}{12}\right) = \frac{65}{32} Amps$$

So $\quad I = I' + I'' + I''' = \left(\frac{5}{2}\right) + \left(-\frac{5}{8}\right) + \left(\frac{65}{32}\right) = \frac{35}{8} Amps$

**Thevenin's theorem:**

"Any linear two terminal circuits can be replaced by an equivalent network consisting of a voltage source ($V_{Th}$) in series with a resistance ($R_{Th}$)."

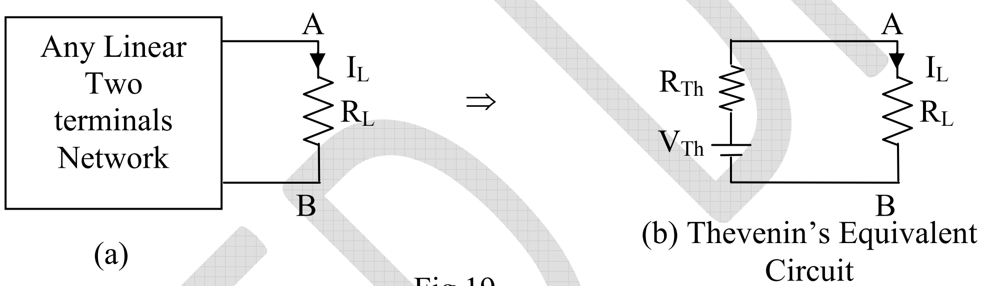



Fig.19

Where

$V_{Th}$ = Open circuit voltage at load terminals

$R_{Th}$ = Equivalent resistance of the network at load terminals when the sources are made in-operative.

And $\quad I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$

**Example 10:** Solve example 5 by Thevenin's Theorem.

**Solution:**

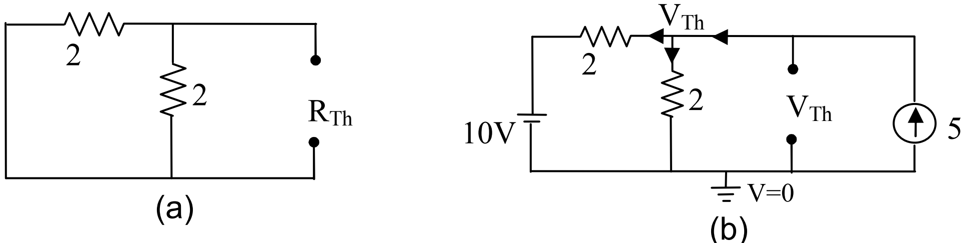



Fig.20

*Find $R_{Th}$* (Fig.20a)

$$R_{Th} = (2 \parallel 2) = 1\Omega$$

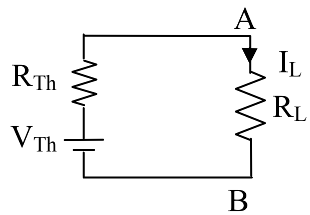
*Find $V_{Th}$* (Fig.20b)

Apply KCL at node $V_{Th}$

$$\left(\frac{V_{Th} - 10 - 0}{2}\right) + \left(\frac{V_{Th} - 0}{2}\right) = 5$$

$V_{Th}$ = 10 Volts

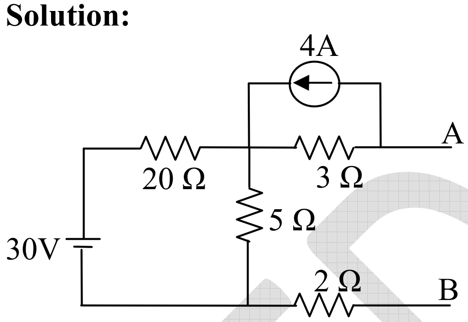
So $I_L = \dfrac{V_{Th}}{R_{Th} + R_L} = \dfrac{10}{1+4} = 2\ Amps$



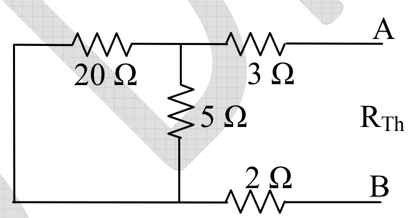
(c) Thevenin's Equivalent Circuit

**Example 11:** Find Thevenin's equivalent circuit across AB of following circuit (Fig.21a).

**Solution:**



(a)



(b)

*Find $R_{Th}$* (Fig.21b)

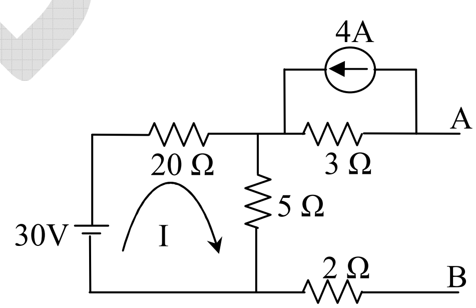
$$R_{Th} = (20 \parallel 5) + 3 + 2 = 9\Omega$$

*Find $V_{Th}$*(Fig.21c)

$$V_{Th} = -3 \times 4 + 5I$$

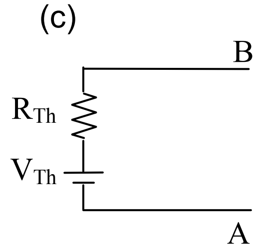
$$V_{Th} = -3 \times 4 + 5\left(\frac{30}{20+5}\right)$$

$$V_{Th} = -6\ Volts$$



(c)

So Its Thevenin's equivalent circuit is shown in fig.21d, It is to be noted here that terminal B is +Ve and A is –Ve.

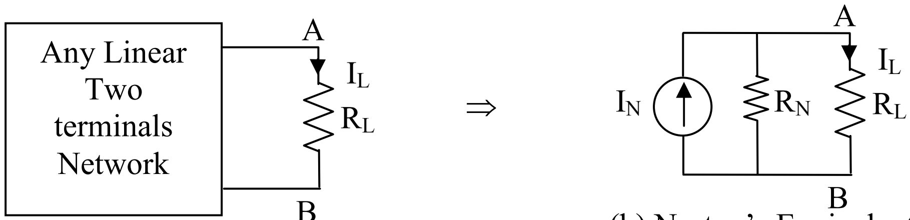


(d) Thevenin's Equivalent Circuit

**Norton's theorem:**

Fig.21

"Any linear two terminal circuits can be replaced by an equivalent network consisting of a current source ($I_N$) in parallel with a resistance ($R_N$)."




(a)

(b) Norton's Equivalent Circuit

Fig.22

Where

$I_N = I_{sc}$ = Short circuit current at load terminals

$R_N$ = Equivalent resistance of the network at load terminals when the sources are made in-operative ($=R_{Th}$).

And $$I_L = \frac{R_N}{R_N + R_L} I_N$$

**Example 12:** Solve example 5 by Norton's Theorem.

**Solution:**

*Find $R_N$* (Fig.22a)

Same as $R_{Th}$ $\quad R_{Th} = (2 \| 2) = 1\Omega$

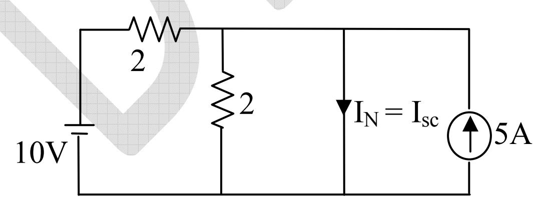
*Find $I_N$* (Fig.22b)

$$I_N = 5A + \frac{10}{2}A$$

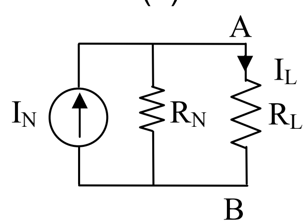
$$I_N = 10\,Amps$$

$$I_L = \frac{R_N}{R_N + R_L} I_N$$

$$I_L = \frac{1}{1+4} 10 = 2\,Amps$$




(a)

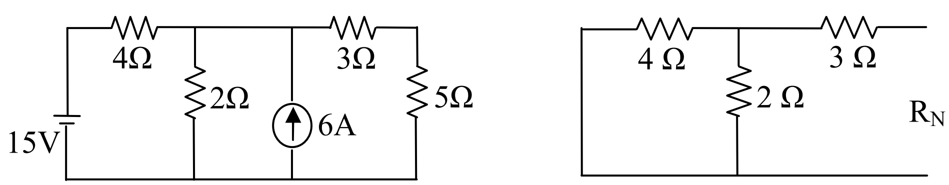



(b) Norton's Equivalent Circuit

Fig.23

**Example 13:** Find current and voltage across 5 ohm resistance by Norton's theorem (Fig.24a).

**Solution:**




(a) (b)

*Find $R_N$* (Fig.24b)

$$R_N = (4 \parallel 2) + 3 = \frac{13}{3}\Omega$$

*Find $I_N$* (Fig.24c)

Apply KCL at node $V_1$

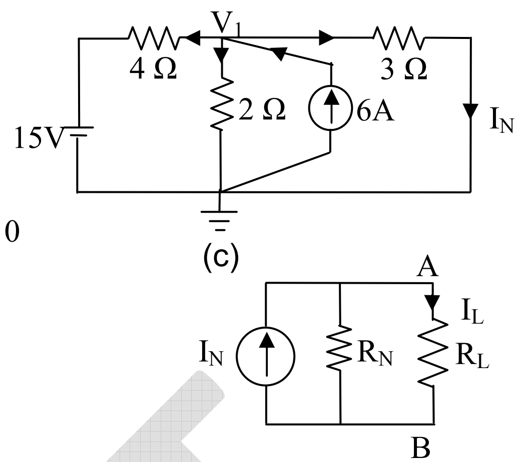
$$\left(\frac{V_1 - 15 - 0}{4}\right) + \left(\frac{V_1 - 0}{2}\right) - 6 + \left(\frac{V_1 - 0}{3}\right) = 0$$

$$V_1 = \frac{117}{13} Volts$$

$$I_N = \left(\frac{V_1 - 0}{3}\right) = 3 Amps$$

$$I_L = \frac{(13/3)}{(13/3) + 5} 3 = \frac{13}{9} Amps$$

$$V_L = I_L R_L = \frac{13}{3} Volts$$




(d) Norton's Equivalent Circuit

Fig.24

**Example 14:** Find Norton's equivalent circuit at A-B of example 11.

**Solution:**

*Find $R_N$* (Fig.21b)

$$R_N = R_{Th} = (20 \parallel 5) + 3 + 2 = 9\Omega$$

*Find $I_N$* (Fig.25a)

Apply Node analysis at node $V_1$ after applying source transformation

$$\left(\frac{V_1 - 30 - 0}{20}\right) + \left(\frac{V_1 - 0}{5}\right) + \left(\frac{V1 - 12 - 0}{3}\right) = 0$$

$$V_1 = \frac{76}{3} V$$

$$I_N = \left(\frac{V_1 - 12 - 0}{20}\right) = \frac{2}{3} A$$

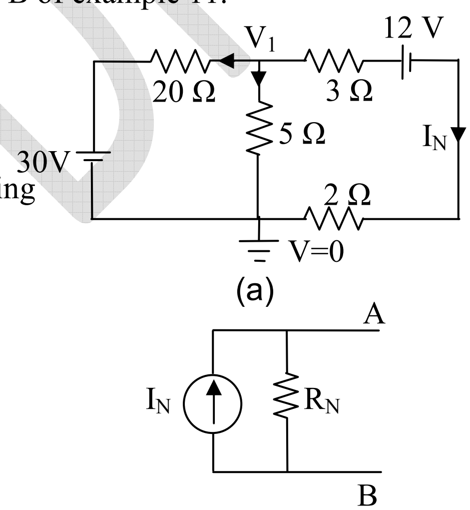
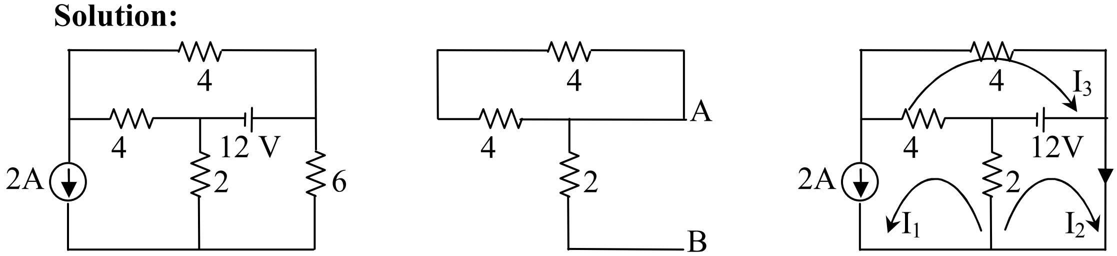



Fig.25 (b) Norton's Equivalent Circuit

**Example 15:** Use Norton's theorem to find out current in 6 ohm resistance and verify it with Thevenin's theorem (Fig.26a).

**Solution:**




(a) (b) (c)

**By Norton's Theorem:**

*Find $R_N$ (fig.26b)*

$R_N$=2 Ohms

*Find $I_N$ (fig 26c)*

Clearly from mesh 1

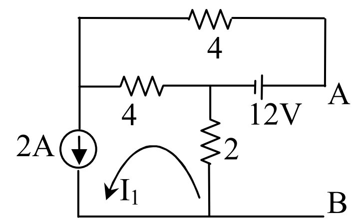
$I_1 = 2$ A

Apply KVL in mesh 2

$2(I_2+I_1)-12=0$

So $I_2 = I_N = 4$ Amp

$$I_L = \frac{2}{2+6}4 = 1\,Amps$$




(d)

Fig.26

**By Thevenin's Theorem:**

*Find $R_{Th}$ (fig.26b)*

$R_{Th}$ = $R_N$=2 Ohms

*Find $V_{Th}$ (fig 26d)*

Clearly from mesh 1

$I_1 = 2$ A

So $V_{Th} = 12-2I_1= 8$ Volts

$$I_L = \frac{8}{2+6} = 1\,Amps$$

Hence verified

**Maximum Power Transfer Theorem:**

"Maximum power is transferred by a circuit to a load resistance ($R_L$), when $R_L$ is equal to Thevenin's equivalent resistance ($R_{Th}$) of that network."

So for maximum power

$R_L = R_{Th}$

And maximum power will be

$$P_{\max} = \frac{V_{Th}^2}{4R_L}$$

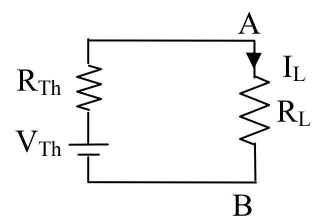



Fig.27 Thevenin's Equivalent Circuit

**Proof:**

Load current will be

$$I_L = \frac{V_{Th}}{R_{Th}+R_L} \text{------} (1)$$

Power

$$P = I_L^2 R_L$$

$$= \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

$$= V_{Th}^2 \frac{R_L}{(R_{Th}+R_L)^2} \text{------} (2)$$

Differentiating equation equa (2) w. r. t. $R_L$ and put $dP/dR_L = 0$

$$\frac{dP}{dR_L} = V_{Th}^2 \left\{ \frac{(R_{Th} + R_L)^2 \times 1 - R_L \times (R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right\} ------(3)$$

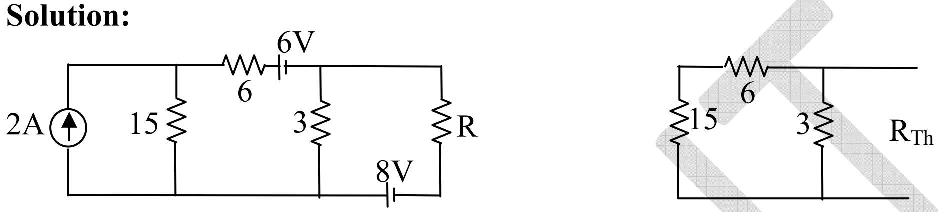
$R_L - R_{Th} = 0$

Or $\quad R_L = R_{Th} \qquad$ Put this in equation (2)

$$P_{max} = \frac{V_{Th}^2}{4R_L}$$

**Example 16:** Find out the value of R for maximum power transfer to this load and find out the value of maximum power (fig.28a).

**Solution:**

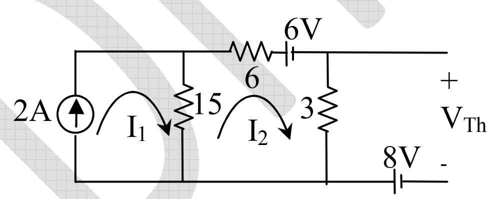



(a) (b)

*Find the $R_{Th}$ (Fig.28b)*

$$R_{Th} = (15+6) \| 3 = \frac{21}{8}\Omega$$

*Find $V_{Th}$ (Fig.28c)*




(c)

Clearly $\qquad I_1 = 2$ A

By KVL in mesh 2

$+6I_2 + 6 + 3I_2 + 15(I_2 - I_1) = 0$

So $\quad I_2 = 1$ A

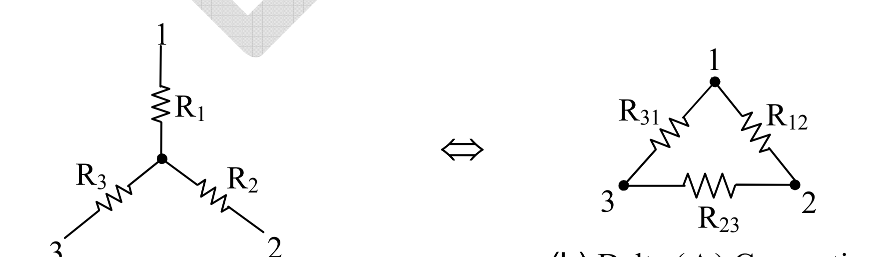
$V_{Th} = 3I_2 + 8 = 11 Volts$

Hence for maximum power transfer to R

$R_L = R_{Th} = \frac{21}{8}\Omega \qquad$ and $\qquad P_{max} = \frac{V_{Th}^2}{4R_L} = 11.524 Watt$

Fig.28

**Star-Delta and Delta-Star transformation:**




(a) Star or WYE (Y) Connection

(b) Delta ( $\Delta$ ) Connection

Fig.29

| Star to Delta | Delta to Star |
|---|---|
| $R_{12} = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$ | $R_1 = \dfrac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$ |
| $R_{23} = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$ | $R_2 = \dfrac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$ |
| $R_{31} = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$ | $R_3 = \dfrac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$ |

**Proof:**

$$(\text{Equivalent resistance at } 1 - 2)_Y = (\text{Equivalent resistance at } 1 - 2)_\Delta$$

$$R_1 + R_2 = (R_{23} + R_{31}) \| R_{12}$$

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} - - - - - (1)$$

Similarly

$$(\text{Equivalent resistance at } 2 - 3)_Y = (\text{Equivalent resistance at } 2 - 3)_\Delta$$

$$R_2 + R_3 = (R_{12} + R_{31}) \| R_{23}$$

$$R_2 + R_3 = \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} - - - - - (2)$$

$$(\text{Equivalent resistance at } 3 - 1)_Y = (\text{Equivalent resistance at } 3 - 1)_\Delta$$

$$R_3 + R_1 = (R_{12} + R_{23}) \| R_{31}$$

$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} - - - - - (3)$$

**Delta to star**

Equation (1) + (2) + (3)

$$2(R_1 + R_2 + R_3) = \frac{2(R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12})}{R_{12} + R_{23} + R_{31}}$$

$$R_1 + R_2 + R_3 = \frac{R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12}}{R_{12} + R_{23} + R_{31}} - - - - - (4)$$

(4) - (2) $\quad R_1 = \dfrac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} - - - - - (5)$

(4) - (3) $\quad R_2 = \dfrac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}} - - - - - (6)$

(4) – (1) $\quad R_3 = \dfrac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} - - - - - (7)$

**Star to Delta**
**From Equation (5), (6) & (7)**

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}\left(R_{12} + R_{23} + R_{31}\right)}{\left(R_{12} + R_{23} + R_{31}\right)^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}}{\left(R_{12} + R_{23} + R_{31}\right)} -----(8)$$

(8) / (7) $\qquad R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$

(8) / (5) $\qquad R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$

(8) / (6) $\qquad R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$

**Example 17:** Find the equivalent resistance between the terminals a-b of the bridge circuit of the fig.30a

**Solution:**

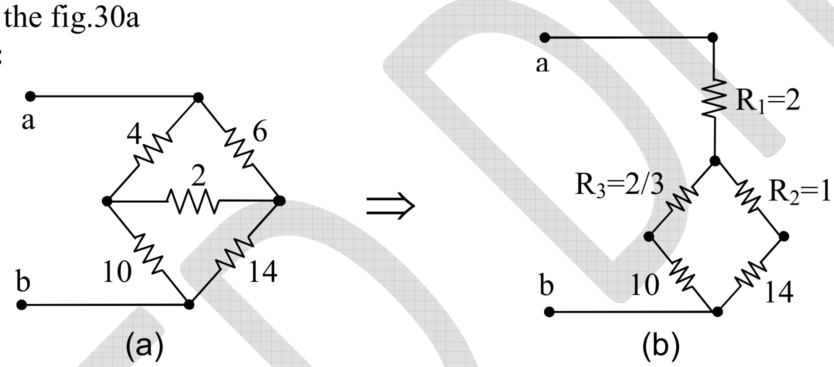



Fig. 30

**Apply delta to star transformation**

$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{6 \times 4}{12} = 2\Omega$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{6 \times 2}{12} = 1\Omega$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{4 \times 2}{12} = \frac{2}{3}\Omega$$

$$R_{ab} = 2 + \left[\left(1 + 14\right) || \left(\frac{2}{3} + 10\right)\right]$$

$$R_{ab} = 8.234\Omega$$