Objective: To determine response of first order and second order systems for step input for several of constant `K' using linear simulator unit and compare theoretical and practical results.

Apparatus Used:

Equipment Description

Signal Sources:

(a) Square Wave : Frequency 40-90 Hz (variable): p-p amplitude 0-2 volt (variable)
(b) Triangular : Frequency 40-90 Hz (variable): p-p amplitude 0-2 volt (variable)
(c) Trigger : Frequency 40-90 Hz (variable): Amplitude ± 5 volt (approx)

THEORY: A thorough understanding of transient response analysis is a pre-requisite for this experiment. Although the topic is covered in great detail in all the text books on automatic control, a brief description of the portions relevant to the experiment is presented below.

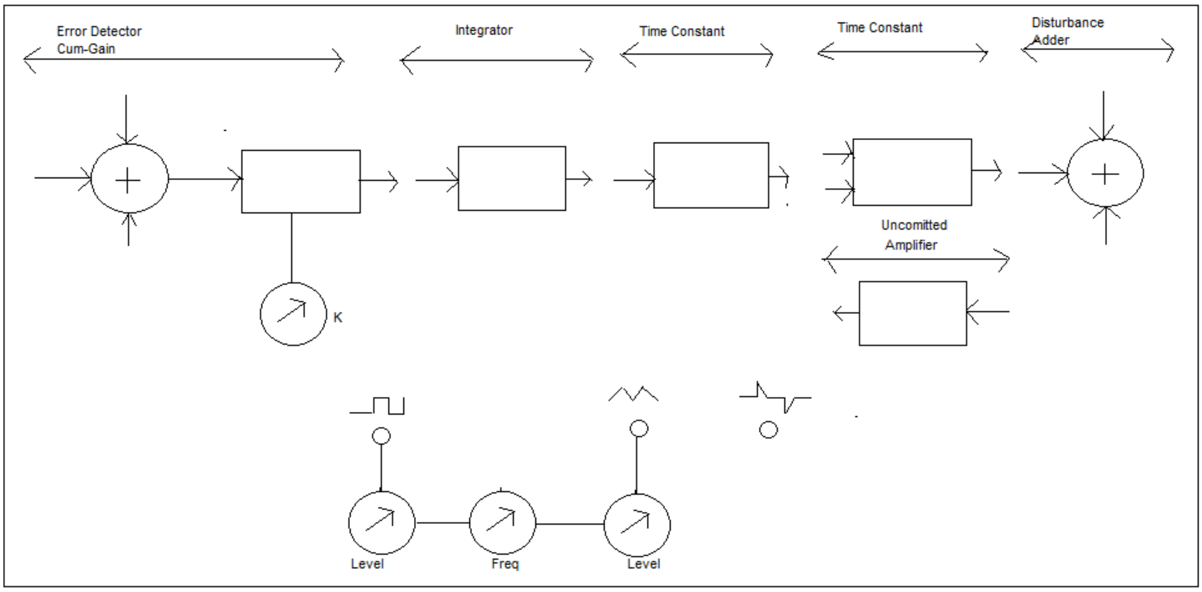

Fig1. Linear System Simulator

1. **First Order System:** These are characterized by one pole and/or zero. A pure integrator and a single time constant, having transfer function of the form K/s and K/ (sT+1), are the two commonly studies representatives of this class of system. Many thermal systems thermal systems and electrical systems with R-C/R-L element are the examples of first order systems.

   Unit step response of the systems are computed as follows and are shown in fig 2(a)

   If C(s)/R(s)=G(s)= K/s then for R(s)=1/s

   $C(s)=K/S^2$, and c(t)=K*t ………………………………………………………..(1)

   Again if G(s)=K/(sT+1) then with R(s)=1/s

   C(s)=K/s(sT+1), and $c(t)=K(1-e^{-t/T})$………………………………………......(2)

   Time constant of the system is defined from eq(2) at t=T which gives

   $C(T)=K(1-e^{-1})=0.632K$

This is an important characteristic of the system which is also defined in terms of the slope of the response curve at t=0

For proper viewing in CRO, the step input needs to be replaced by a square wave of sufficiently low frequency (to allow c(t) in equation 2 to reach up to 99% of its final value). This is shown in the second sketch of fig 2(b). However in the first sketch of fig 2b a triangular wave output results since frequency is not sufficiently low. If may further be seen that if the square wave is of frequency f and peak-to-peak input amplitude is 1V, the peak-to-peak amplitude of the triangular wave at the output of the pure integrator is given as K/4f.

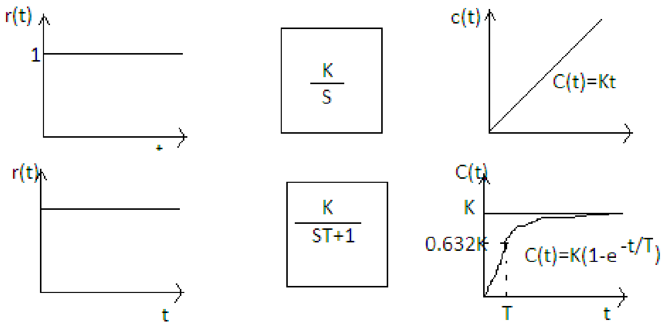

Fig 2(a) Unit Step Response of First Order Transfer Functions

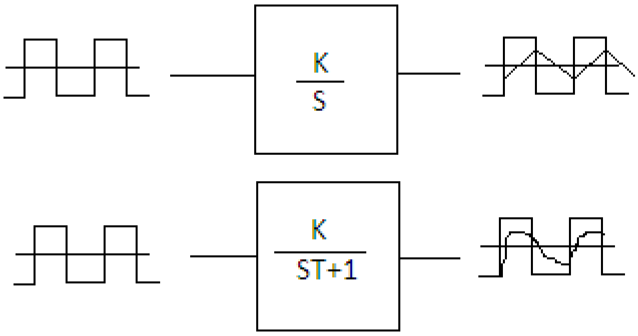

Fig 2(b) Unit Step Response of First Order Transfer Functions

2. Second order System:

These systems are characterized by two poles and up to two zeros. For the purpose of transient response studies, zeros are usually not considered primarily because of simplicity in calculations and also because the zeros do not affect the internal modes of the system. A great deal of analytical results regarding second order systems is available in the text books. This forms the basis of studying higher order systems many of which can be approximated to second order.

A second order systems is represented in the standard form as

$$G(s) = \frac{w_n^2}{s^2 + 2\delta w_n s + w_n^2}$$

Where δ is called the damping ratio and w_n the undamped natural frequency. Depending upon the value of δ, the poles of the system may be real, repeated or complex conjugate which is reflected in the nature of its step response. Results obtained for various cases are:

(a) Under damped case (0<δ<1)

$$c(t) = 1 - \frac{e^{-\delta\omega_n t}}{\sqrt{(1-\delta^2)}} . \sin(\omega_d t + tan^{-1}\frac{\sqrt{(1-\delta^2)}}{\delta}) \qquad (3)$$

Where $\omega_d = \omega_n\sqrt{(1-\delta^2)}$ is termed the damped natural frequency. A sketch of the unit step response for various values of δ is available in the text books.

(b) Critically damped case ($\delta = 1$)

C(t)= 1-$\omega^{-\omega_n t}$(1+$\omega_n t$) (4)

(c) Overdamped case ($\delta > 1$)

C(t)= 1+ $\frac{\omega_n}{2\sqrt{(\delta^2-1)}}(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{2})$ (5)

Where $s_1 = \left(\delta + \sqrt{(\delta^2 - 1)}\right)\omega_n$ and $s_{2=} \left(\delta - \sqrt{(\delta^2 - 1)}\right)\omega_n$

Closed Loop Systems

Closed loop or feedback systems involve a measurement of the output of the system and generation of the control signals, which are based on decision making under the influence of a command or reference and the measured values of the output (Fig.3). Such system is of great interest to control engineers due to features like automatic correction, disturbance rejection, immunity to noise and parameter variation etc. A study of the performance of closed loop system is the basics objective of this experiment. It may be easily appreciated that although the mathematically description of a closed loop system is no different form that discussed earlier, the fact that variation of forward path gain shifts the pole location, changes the situation drastically and makes direct computation of little value. Referring to Fig.3, the closed loop transfer function for different open loop functions are shown below:

(1) For G(s) = $\frac{K}{s}, \frac{C(s)}{R(s)} = \frac{K}{s+K} = \frac{1}{s/_{K+1}}$

This gives a step response similar to Eq. (2) with time constant decreasing as K increase.

(2) For G(s) = $\frac{K}{sT+1}, \frac{C(s)}{R(s)} = \frac{K}{sT+K+1} = \frac{K/_{(1+K)}}{sT/_{(1+K)+1}}$

This has a step response similar in nature as obtained above.

(3) For G(s) = $\frac{K}{s(sT+1)}, \frac{C(s)}{R(s)} = \frac{K}{s^2T+s+K} = \frac{K}{s^2+s/_{T}+K/_{T}}$

Which gives response similar to Esq. (3), (4) or (5) depending upon the value of K.

Thus the response of the closed loop system can be altered by varying the open gain K and as a consequence it should be possible to choose K to obtain a suitable performance.

This leads to the concept of performance characteristics as defined on the step response of an underdamped second order system in Fig. 4. It must be noted that these specification are not restricted to second order system, although the mathematical expression/definitions given below are valid and computationally practicable for second order systems only.

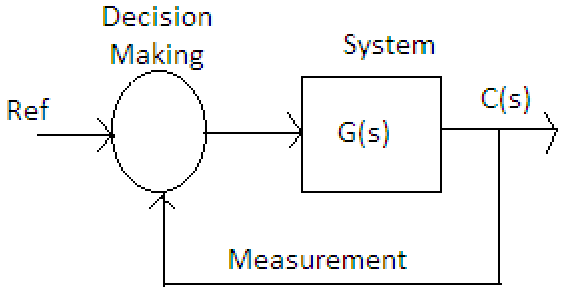

Fig3 Block Diagram of A closed Loop System

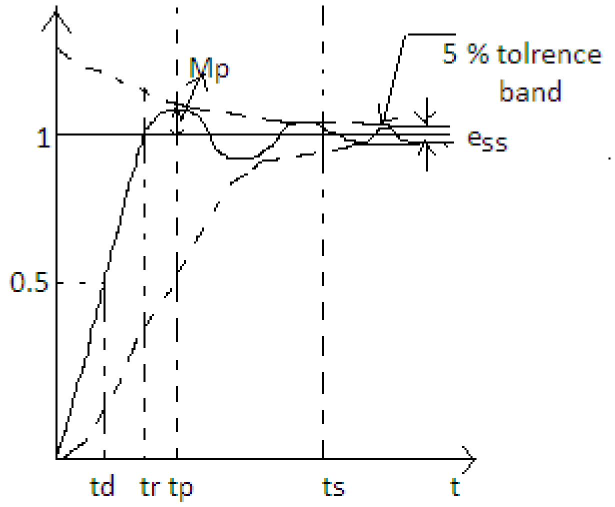

Fig 4 Step Response of an under damped System

Procedure:

Open Loop Response:

As a first step, the open loop transfer function of all the books viz. integrator, time constant, uncommitted amplifier and error detector/adders are to be determined experimentally. All measurement is done with the help of a measuring oscilloscope and the signal source is the built -in –square wave generate in each case. Further, to get a properly synchronized waveform, especially for small values of signals, it will be convenient to use the built – in trigger source keeping the CRO in external triggering mode. A double beam CRO for the simultaneous viewing of input output is recommended. Note that the value of $k_{1,}K_{2,}K_3$ and $T_{1,}T_2$ obtained experimentally may differ somewhat from their nominal value indicated.

(a) Error Detector Cum Variable Gain

- Apply a 100 mV square wave signal to any of the three inputs.
- Set the gain setting potentiometer to 10.0

- Measure the P-P output voltage notes its sign. Calculate the gain. This is the maximum value of gain possible for this block.
- Repeat for the other two inputs one by one.
- Write the equation of this block and verify by connecting the signal to all three inputs.

(b) Disturbance Adder

- This section may be tested exactly in the same manner as (a) except that three are only two inputs and there is no gain setting potentiometer.

(c) Uncommitted Amplifier

- **A**pply a 1 volt p-p square wave input.
- Measure the p-p output voltage and note its sign.
- Record the equation of this block for later use.

(d) Integrator

- Apply a 1 volt p-p square wave input of known frequency (frequency measured by CRO)
- Measure the p-p output voltage of the triangular wave and also note its phase.
- Calculate the gain constant K of integrator as discussed, and write the transfer function of this block.

(e) Time Conctant

- Apply a 100 mV square wave of known frequency (measured by the CRO). For this experiment, the frequency should be selected towards the lower and to ensure that steady state is nearly reached.
- Find on the track the time t=T at which the response reaches 63.2%. this is the time constant.
- Find on the track the steady state value of the response. The value of K is given by the ratio of p-p stead state output to the p-p input amplitude.
- Write transfer function of the block as discussed.
- The wave form in the CRO may be traced on the tracing paper for analysis

Closed Loop Response – First Order System

- Two forms of first order closed loop system, as shown in Fig.5 are possible. Make proper connections for the configuration chosen.
- Apply a 1 volt square wave input and trace the output waveform on a tracing paper for K = 0.5. 1.0, 1.5 …. Calculate the time constant in each case and compare with theoretical result.

- Also calculate the steady state errors for the cases and compare with theoretical results.
- If the open loop transfer function in the chosen configuration was of type-I, the steady state error above would be zero for a step input. To find steady state error for ramp input, apply a 1 volt p-p triangular wave input. Keeping the CRO in X-Y mode, connect input to the X input and the system

output to the Y input. A trace as shown in fig 6 will be seen on the CRO in which the vertical displacement between the two curves is the steady state error.

Closed Loop Response-Second Order System:

- Choose and wire a suitable second order system configuration from fig 7.
- Apply a 1 volt p-p square wave input and trace the output waveform on tracing paper for different values of K. Obtain peak percent overshoot, settling time, rise time and steady state errors from the tracing and calculate δ and w_n. Compare with theoretical results.

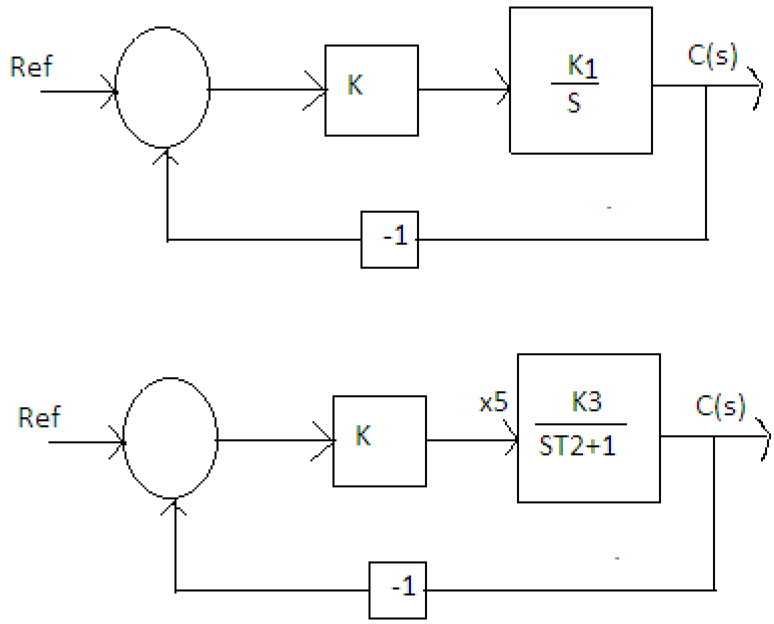

Fig5: Closed Loop Options for First Order Systems

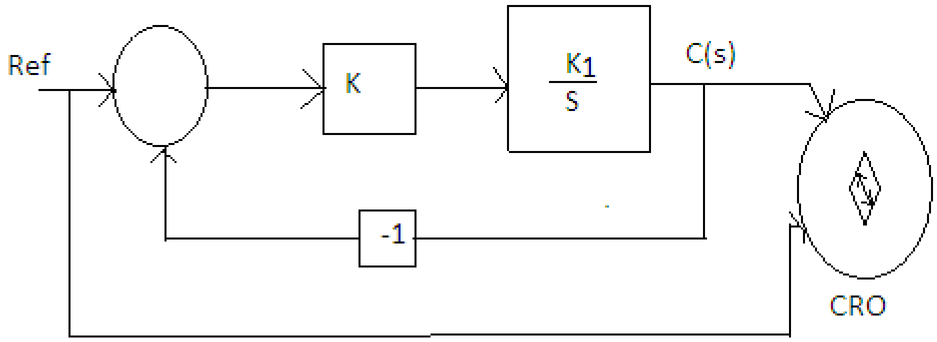

Fig6: Steady-State Error for Ramp Input

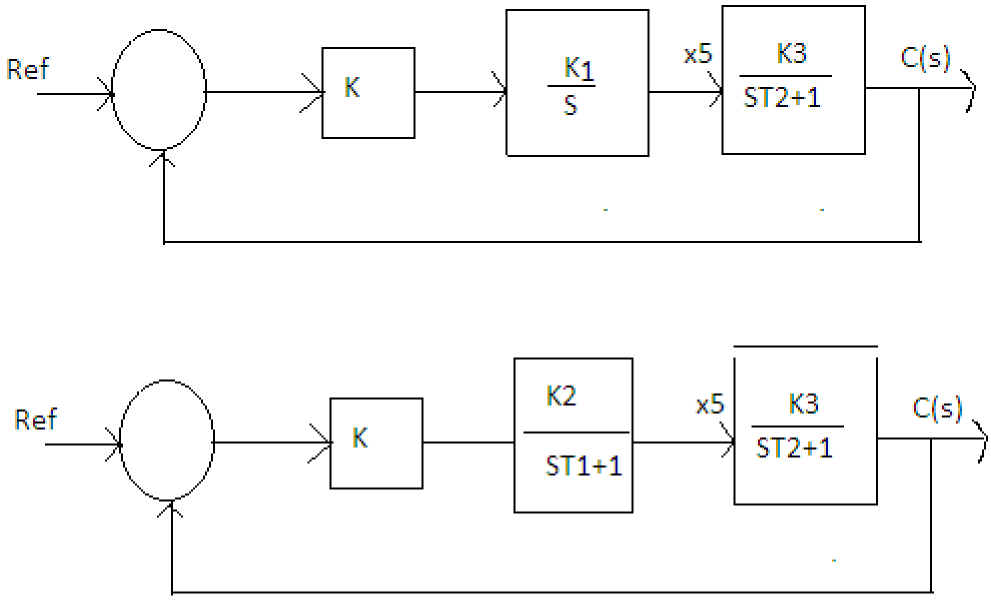

Fig7: closed Loop Options for Second Order systems

OBSERVATIONS:

S.No.	K	Mp	tr	tp	ts	δ^{calc}	w_n^{calc}	δ^{the}	w_n^{th}

RESULT:

The calculated and theoretical values are nearly equal.

PRECATIONS:

1. Do not operate system without a supervisor.
2. Connect appropriate signal.