Objective:

To study the performance characteristics of an analog PID controller using simulated systems.

Apparatus Used:

Name of the apparatus	Range/Rating	Quantity
1. PID System		1
Proportional Gain k_c:	0 to 20	
Integral time constant t_i:	5 – 100 msec	
Derivative time constant t_d:	0- 20 msec	

Theory:

The performance of a physical system is not always good enough for a given application. In such a situation the characteristics of the system needs to be modified. This is referred to as "compensation design". Standard procedure available for compensation include time and freq domain designs of a variety of compensation networks .such design methods have been successfully used in many practical dynamic control systems.

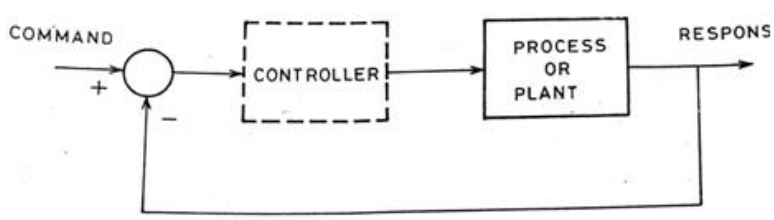


FIG. 1 BLOCK DIAGRAM OF THE SYSTEM

The performance of the system is evaluated in terms of a set of performance specifications e.g. rise time, peak time, settling time, peak percent overshoot and steady and steady state error in the time domain and gain margin phase margin, closed loop bandwidth etc. in the frequency domain.

Another approach towards improving the performance of systems has been through elementary control actions called control terms- inserted in the forward path of an existing control system. The block diagram of fig 1 shows the location of such a controller in a unity feedback system. The controller work comprises two or three of the following control terms:

a) Proportional, P
b) Integral, I
c) Derivative, D

The resulting control system may turn out to be a PI, PD or PID controller.

The two and three term controllers indicated above have been used more commonly by process industries e.g. Petroleum, chemical, powder, food etc. for the control of temperature, pressure, flow and similar variable. A common features of these system is their sluggish response which calls for accurate and slow integration and sensitive differentiation. Although near ideal electronic differentiator and integrator circuits are difficult to achieve except with high temperature operational amplifiers and good quality component PI and PD controller valves have existed in the pneumatic and hydraulic environment for a long time.

In the present unit attempt has been made to expose the student to the study and design of PID controller using simulated systems. The speed of response has been deliberately scaled up to have a fast and easy viewing on CRO.

The PID Controller

Structure

The equation of PID Controller is given by

$$m(t) = k_c e(t) + k_i \int e(t)dt + k_d \frac{de(t)}{dt}$$

Where e (t) = error signal
m (t) = PID o/p or plant i/p

k_c=proportional gain
k_i= integral gain
k_d=derivative gain
In the Laplace domain, the above eq is written as

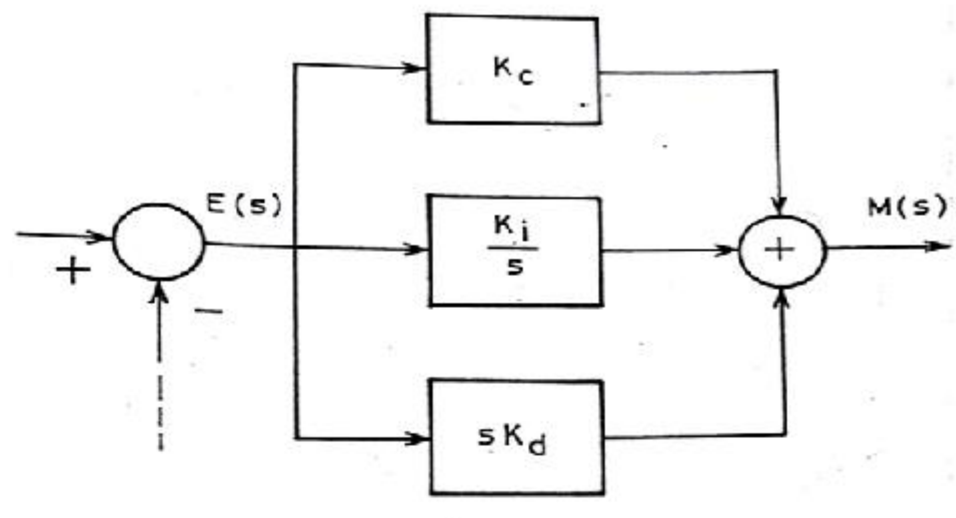


FIG. 2 PID CONTROLLER

M(s) = k_cE(s) + $\frac{k_i}{s}$ E (s) + s k_d E(s)

Which may be represented as the block diagram of fig.3

An alternative representation of the above which is more commonly used I process control literature is as under:

M(s) = k_c (1 + $\frac{1}{T_i s}$ + T_ds) E(s)

Where

$T_i = \frac{k_c}{k_i}$ = Integral time constant

$T_d = \frac{k_c}{k_d}$ =derivative time constant

It is easy to develop the structure of PD, and PI controllers from above , substituting k_i=0 and k_d =0 respectively.

A special terminology used in process control literature is given below to facilitate better understanding.

Proportional Band = $\frac{1}{k_c}$ x 100 %

Reset rate = $\frac{k_i}{k_c} = \frac{1}{T_i}$ per minute

Derivative Time = T_d

In the present unit, the three gains are adjustable in the following range with the help of calibrated 10- turn potentiometers.

k_c: 0 to 20

k_i: 0 to 1000

k_d: 0 to 0.01

Experimental determination of these values are discussed in sec .4

CHARACTERISTICS

From eq (2) the transfer function of the PID controller may be written as G_{PID} (s) = $\frac{M(s)}{E(s)}$ = $\frac{k_d\, s^2 + k_c\, s + k_i}{s}$

$= \frac{k_s}{s}$ (s + w_1) (s + w_2)

Where w_1 and w_2 are the two zeroes of the PID controller transfer function.

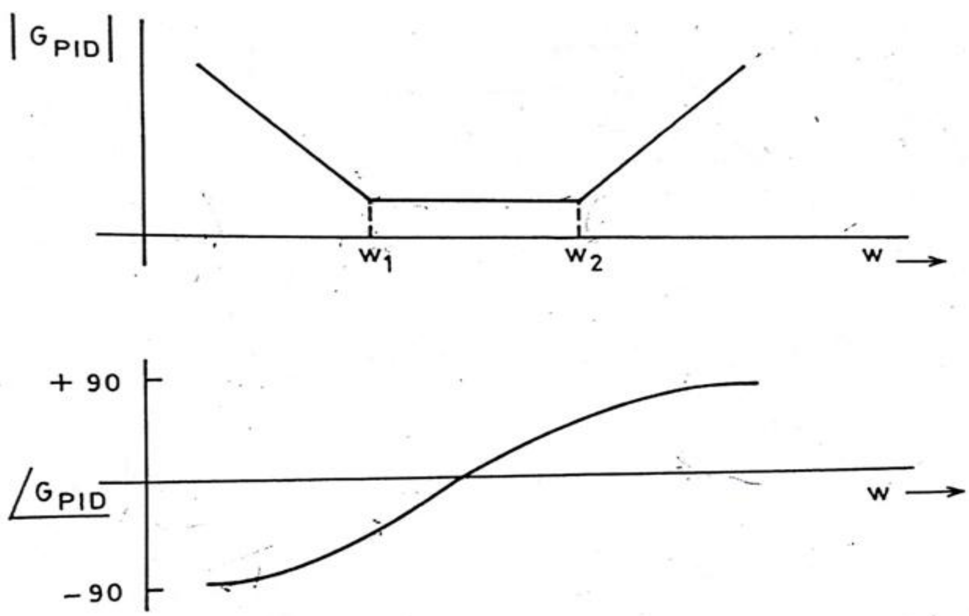


FIG. 3 BODE DIAGRAM OF PID CONTROLLER

The above transfer has a pole at the origin and two real zero for $K_c^2 > 4K_dK_i$.

Notice that a properly designed PID controller should not, in general, have pair of complex conjugate zeroes which may result in reduced damping. Bode diagram of the PID controller is shown in fig. 3.

It may be seen that the controller gain increase without limits as the frequency is decreased. This is due to the integral term, and it results in a reduction of steady state error. However, the negative phase angle introduce by the controller at low frequencies has a destabilizing effect as well. The corner frequency ω_1 should therefore be so located that large negative phase angle occurs at sufficiently low frequencies only, where the plant already has a good stability margin.

Again, the bode diagram of the controller an increased gain at high frequencies accompanied by a positive phase angle. The positive phase angle has a stabilizing effect while the large gain at high frequencies makes the system more responsive to fast or sudden changes. The overall system then becomes relatively more stable, as it capable of taking ‘anticipatory’ action in the presence of signal having fast variation.

Design

The PID controller can be designed both in frequency domain and in the s-plane, through the classical or trial and error design procedure. The method needs the pole zero location or frequency phase response of the plant, for its implementation. A large number of process control systems are however characterized by,

- Incomplete or inaccurate plant questions.
- Extremely slow response.
- Presence of time delays.
- High order transfer function
- Limited possibility of experimentation for identification of the plant, and
- Need for fine trimming the compensator at site

  In such a situation alternative simpler technique of setting the controller parameter (K_c, T_i, T_d), or tuning are of great practical value. Presented below are three technique of tuning a PID controller aimed at obtaining a satisfactory step response of the overall system. Experimental work based on this method.

(a)Trial and error tuning

This is a simple and systematic method for on the line tuning of a PID controller. The method assumes that the three parameters K_c, K_i, K_d are available for adjustment. Following are steps for its implementation:

1. Disconnect or reduce derivative and integral block signals by setting $K_i$and K_d to zero.

. 2. Starting from a low value increase K_c gradually oscillation sets in .this condition is tested by small disturbances generated by varying the reference signal a little. The value by proportional gain so obtained is called ultimate gainK_{cu}.

3. Set K_c to ½ of the value obtained in step 2.

4. Increase K_i gradually until sustained oscillations start again. Set K_i to 1/3 of this value.

5. Increase K_d gradually until sustained oscillations start again. Set T_d to 1/3 of this value.

The above method, although very simple in operation, has the following limitations:

I. A number of systems which are or may be approximated, first or second order transfer functions without time delay do not oscillate. Step 3 is then not possible and the method fails.
II. Open loop unstable systems cannot be handled by this method.
III. Tuning of very slow systems by this method is extremely time consuming.
IV. Sustained oscillation may not be acceptable or may be risky in some physical process such as a large chemical process.

(b) Continuous Cycling Method

In this method given by Ziegler and Nichols the first step is to determine experimentally the value of ultimate gain, K_{cu} as suggested in the previous method. The time period of the resulting sustained oscillations is referred to as ultimate periodP_u. Based on the values of K_{cu} and P_u the controller setting are obtained from Table 1 which as essentially empirical in nature.

Controller Type	K_c	T_i	T_d
P	$0.5K_{cu}$	-	-
PI	$0.45K_{cu}$	$0.833p_u$	-
PID	$0.6K_{cu}$	$0.5p_u$	$0.125p_u$

The values of K_c and K_d may be calculated from eq.3 for implementation on the present system.

Some variation in the coefficient settings have also been suggested by various workers. In any case the above values should be taken as the initial settings and should invariably be followed by the fine tuning via trial and error.

Most of the limitations of the first method are still present in this method. However the continuous cycling method is less time consuming.

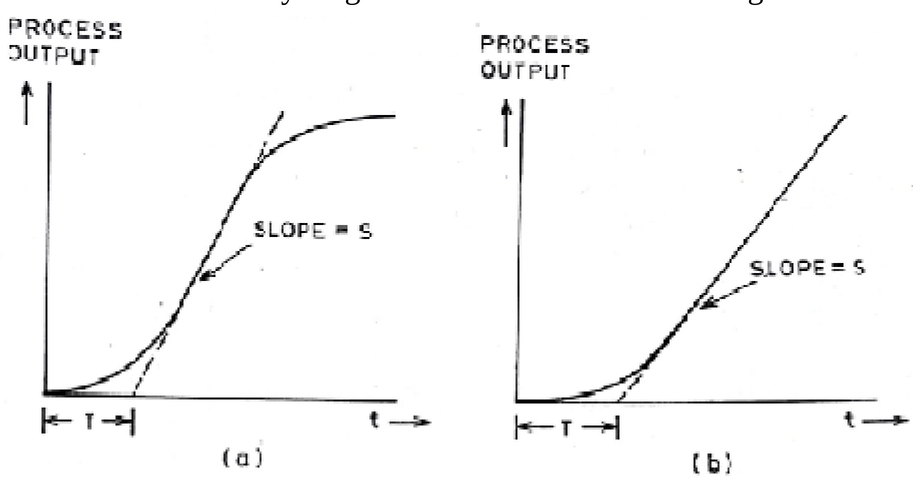


FIG. 4 STEP RESPONSE OF (a) TYPE-0, (b) TYPE-1 SYSTEMS

Process Reaction curve Method

This is a second on line method purposed by Ziegler and Nichols and is very attractive because it is based on a simple experiment. The plant is modelled as a first order function with time delay. The open loop step response of the plant, called reaction curve of the process. Is experimentally obtained. Typical step response for type-0 and higher type number system are shown in fig 4(a) and 4(b) respectively. The step responses are characterized by two parameters,

(I) Slope S of the tangent drawn at the point of inflection, and

(II)Time T at which the tangent intersects the X axis.

The values of S and T are obtained graphically as shown in fig 5. In the input step changes was M the PID parameters are given by the Table below.

Controller Type	K_c	T_i	T_d
P	$\frac{M}{ST}$	-	-
PI	$\frac{0.9M}{ST}$	3.33T	-
PID	$\frac{1.2M}{ST}$	2T	0.5T

Once again, the above values are empirical in nature and therefore fine tuning of the parameters may be needed in specific cases. The values of $K_i$and K_d may be calculated from eq. 2 for implementation on the present unit.
Although the process reaction curve method based on a single experimentation is fast and simple, it does have some limitations as given below:

i. The step response obtains in the open loop may not be satisfactory in case the system is highly nonlinear or open loop unstable.
ii. Accuracy is limited due to the graphical procedure involved.

In conclusion it may be said that any method used to calculate the parameters must be followed by a fine tuning on the operational process.

EXPERIMENTS

A very wide range of experimentation is possible with the unit, however the ones suggested below are aimed at bringing out the feature of PID controller in one or two laboratory classes of usual duration. It may be mentioned that a conventional CRO display has been obtained by a proper design of the system. Tuning of PID controller is therefore very fast and avoids expensive accessories like an X-Y/t recorder.

Experimentation in the following material has been suggested with a system having a time delay block .Such a representation is closer to many real life systems which have pure time delay. However this takes the system closer to instability which can then accept only small values of k_c , k_i etc. As a result the settings of P,I and D controls may be difficult to make for a beginner . In that case it is suggested that the beginner may experiment with a system with one / two time constant blocks without time delay block.

Before starting the experiments, it will be helpful to understand the calibrated dials of P, I and D control knobs. In section 4.1, the student finds the maximum value of k_c , k_i and k_d or in other words the full scale values of these parameters. The potentiometers used are 10 turn types and each turn is divided into 10 parts by the dial scale. Each part is further divided into 5 divisions so that the total dial range of 0 to 1 has a least count of 0.002. A full revolution of a knob corresponds to a change of 0.1 in dial reading. To obtain a parameter value , multiply the dial setting by the corresponding full scale (FSV) for P control is 20 then a dial setting of 0.032 will correspond to a k_c =0.032 x 20 = 0.64.

CONTROLLER RESPONSE

The time domain response of the PID controller is of great value for a good understanding of its performance. This also enables the readers to calibrate the three potentiometers, it felt necessary. The steps suggested are:

1. Apply a square wave signal of 100 mV p-p at the i/p of the error detector. Connect P, I and D outputs to the summer and display controller o/p to the CRO.
2. With P potentiometer set to maximum and I and D potentiometers set to 0 , obtain maximum value of k_c as :

$$\frac{\text{p−p square wave output}}{\text{p−p square wave i/p}} = \frac{\text{p−p square wave o/p}}{0.1}$$

3. With I potentiometer set to maximum and P and D potentiometers set to 0,a ramp will be set on CRO. Maximum value of k_i is given by :

   k_iMax = $\frac{4\ X\ f\ X\ (p-p) triangular\ wave\ output\ \ amplitude\ in\ volts}{p-p\ square\ wave\ amplitude\ in\ volts}$

   $where\ f\ is\ the\ frequency$ Of the input

4. Set D potentiometer to maximum P and I potentiometers to 0. A series of sharp pulses will be seen on the CRO. This is obviously not suitable for calibrating the D potentiometer. Instead applying a triangular wave at the input of the error detector a square wave is seen on the CRO.

   k_dMax = $\frac{p-p\ square\ wave\ output}{4\ X\ f\ X\ (p-p) triangular\ wave\ input}$

   $where\ f\ is\ the\ frequency\ of\ the\ input\ signal.$

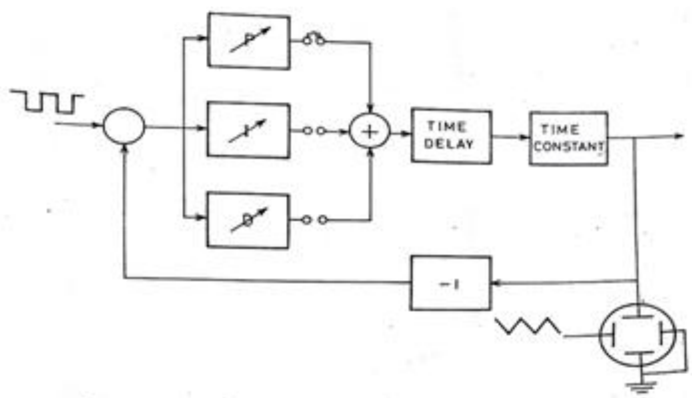


FIG. 6(a) CONNECTION DIAGRAM FOR P- CONTROL

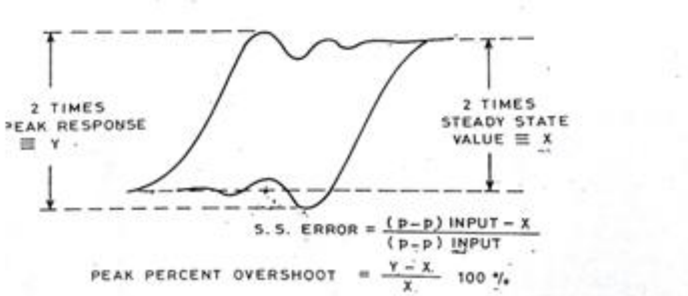


FIG. 5 CRO DISPLAY OF STEP RESPONSE USING TRIANGULAR TIME BASE

Procedure:

Set all the three potentiometers – P, I and D to maximum values and apply a square wave input of 100 mV (p-p). Observe and trace the step response of the PID controller. Identify the effects of the P, I and D controls individually on the shape of this response.

Proportional Control

This experiment would familiarize the student with the present unit and the general features of linear system .The steps suggested are

- Make connections as shown in Fig.6 (a) with process made up of time delay and time constant blocks. Notice that the CRO operation in X-Y mode ensure stable display even at low frequencies.
- Set input amplitude to 1V (p-p), and frequency to a low value.
- For various values of K_c=0.2, 0.4… measure from the screen the values of peak overshoot and steady state error and tabulate (Refer to Fig.6 (b) for graphical calculation).

Alternatively, a simultaneous display of square wave input and system response using a dual trace oscilloscope may be used to get a very clear idea of the transient and steady state performances. Some flickering may however be observed in this case due to the low frequencies involved.

Observe that the second order type-0 system has non-zero steady state error for step input which decreases with increasing K_c while the peak overshoots increases.

The above experiment may be performed for a variety of system with or without time delay. Note that loop phase must be kept as 180*, if necessary by using the uncommitted amplifier of gain = -1, so that the feedback is negative.

Proportional – Integral Control

The integral term results in increasing the system type number by unity and thus cause improvement in steady state performance. To verify the above with step input one starts with a type-0 system having a non-zero error. Introducing PI control with a properly selected value of Ti should reduce the error to zero. The steps suggested are as follows:

- Make connections for a 1st order type-0 system with time delay (Fig. 6(a) with proportional and integral blocks connected).
- Set input amplitude to 1V (p-p), frequency to a low value and K to zero.
- For K_c=0.6(say), observe and record the peak overshoot and steady state error.
- With the Kc as above, increase Ki in small steps and record peak overshoot and steady state error.

Observe that for a given value of K_c increasing of integral gain K_i improves the steady state performance. Excessive increase of K_i, however results in an inferior transient response

For a detailed study of proportional control, our experimental control, our experimental set-up "Linear System Simulator" is recommended.

Proportional-Integral-Derivative Control

This experiment will demonstrate the improvement in transient performance by the introduction of derivative control. The following steps suggested:

- Make connections as shown in figure 6(a) with proper integral and derivatives blocks connected.
- Set the input amplitude to 1V (p-p), frequency a low value, K_c=0.6, K_i=54.85 and K_d=0.
- The system shows a fairly large overshoot. Record the peak overshoot and steady state error.
- Repeat the above step for a few non-zero values of K_d

Observe the improvement in transient performance with increasing values of T_d, while the steady state error remains unchanged.

- For K_c =0.6, adjust K_i and K_d by trial and error to obtained the best overall response. Record the values of K_c, T_i and T_d . Repeat for K_c=0.4, 0.2 etc.

PID Design by Process Reaction Curve Method

In this experiment the PID parameters are designed by the method of Ziegler and Nichols outlined in sec. 3.23(c). The Unit step response of the open loop system is obtained first. Subsequent steps are:

- Compute S and T from The CRO screen as indicated in fig.(a).
- Calculate the parameters of PID controller from Table 2 which are reproduced below:

$$K_c=\frac{12}{ST}$$

$T_i = 2T$, or $K_i = \frac{0.6}{ST^2}$

$T_d = 0.5T$, or $K_d = \frac{0.6}{S}$

- Set the PID parameters as calculated above and the observe the response. Comment.
- Attempt fine tuning parameters to get a better response.

  All the above mentioned experiments may be carried out on a variety of plants of different orders and type numbers depending on the time allotted in the curriculum.

RESULTS:

Calibration

The calibration results here corresponded to the measurements suggested in section 4.1

a) P Control

Input: Square wave of amplitude 0.1 V (p-p)
Output: Square wave of amplitude 2.0 V (p-p)
K_c (max.): 2.0/0.1 = 20

b) I Control

Input: Square wave of amplitude 0.1 V (p-p)
Time period: 70 msec
Frequency: 1000/70 = 14.286 Hz
Output: Triangular wave of amplitude 1.6 V (p-p)
K_i (Max.) = 4*14.286*1.6/0.1 = 914.2/sec

c) D Control

Input: Triangular wave of amplitude 0.84 V (p-p)
Time period: 70 msec
Frequency: 1000/70 = 14.286 Hz
Output: Square wave of amplitude 0.5 V (p-p)
K_i (max.) = 05/4*14.286*0.84 = 0.0104 sec

PI Control

Input: 1V (p-p) Square wave of low frequency
$K_c = 0.6$
System = type 0 with the time delay fig 6(a)

Scale Reading	K_i (per sec.)	X=2*Steady state value	Y=2*peak response	Steady state error	% overshoot
0.00	0.00	0.50	0.56	0.50	12.00
0.02	18.28	0.60	0.64	0.40	6.67
0.04	36.57	0.72	0.76	0.28	5.55
0.06	54.85	0.84	0.96	0.16	14.28
0.08	73.14	0.92	1.16	0.08	26.08

This system, due to the presence of time delay block, has a greater tendency to become unstable. Readings are therefore restricted to small values of Kc ad Ki. System without time delay will operate satisfactory over wide range of gain values and are recommended in the initial stages of experimentation.

PID Control

Input: 1V (p-p) Square wave of low frequency
$K_c = 0.6$
$K_i = 0.606*914.2 = 54.85$/sec.
System = type – 0 with time delay

Scale Reading	K_d (sec.)	X=2*Steady state value	Y=2*peak response	Steady state error	% overshoot
0.00	0	0.8	0.94	0.2	17.5
0.05	$.52*10^{-3}$	0.8	0.88	0.2	10.0
.10	$1.04*10^{-3}$	0.8	0.84	0.2	5.0
0.15	$1.56*10^{-3}$	0.8	Overdamped (no overshoot)	0.2	-

The trial and error value for best performance (of the system shown in fig. 6(a) in the prototype for Kc = 0.4 were found as:

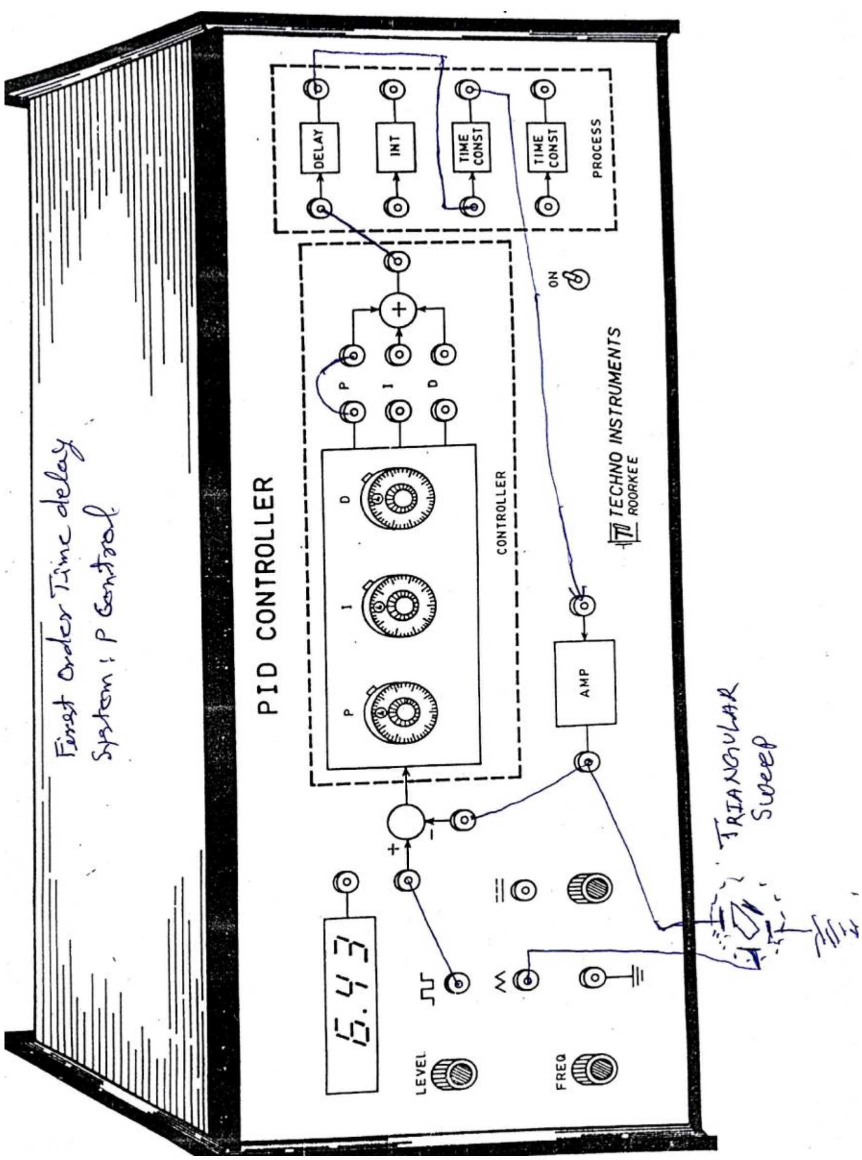
K_i = 0.065*914.2 = 59.423 per sec.

K_d = 0.08*0.0104 = $0.832*10^{-3}$ sec.

Precaution:

1. Do not increase the range of P, I and D randomly.
2. Connect the terminals properly.

.



DELAY
INT
TIME CONST
TIME CONST
PROCESS
ON
P
I
D
CONTROLLER
PID CONTROLLER
TECHNO INSTRUMENTS
ROORKEE
AMP
LEVEL
FREQ
First Order Time delay
System ; P Control
TRIANGULAR
SWEEP