Electrical Resonance

Resonance is defined as the condition in a circuit containing at least one inductor and one capacitor when the supply voltage & supply current are in same phase.

At resonance

i. Angle between V & I is zero

ii. $\phi=0 \Rightarrow \cos\phi=1$, Unity power factor

iii. If Z and Y are given in rectangular form, their imaginary part will be zero i.e.

If $Z=R\pm jX \quad \Rightarrow Im(Z)=0 \Rightarrow X=0$

If $Y=G\pm jB \quad \Rightarrow Im(Y)=0 \Rightarrow B=0$

iv. If Z and Y are given in polar form, their angle will be zero i.e.

If $Z = |Z|\angle\phi \Rightarrow \phi = 0$

If $Y = |Y|\angle\phi \Rightarrow \phi = 0$

Types of resonance: Two types

1. **Series Resonance:** Consider the following circuit

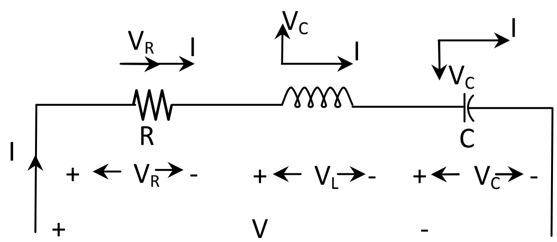

Fig. 1a

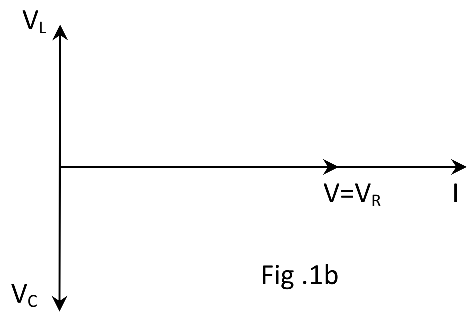

Fig .1b

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

If $X_L = X_C$ then

i. Total reactance $X = X_L - X_C = 0$

ii. Total impedance $Z = R = Z_r$(say)

iii. Current $I = \frac{V}{Z} = \frac{V}{R}$ maximum

iv. Angle between V & I is zero $\Rightarrow \phi=0 \Rightarrow \cos\phi=1$

This condition is called as series resonance and frequency at which it occurs is called resonance frequency (f_r).

So at resonance

$$X_L = X_C \qquad \Rightarrow \omega_r L = 1/\omega_r C \qquad \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \qquad \Rightarrow f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

2. **<u>Shunt Resonance:</u>** Consider the following parallel circuit

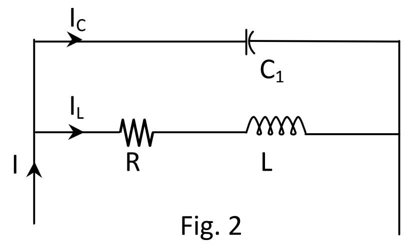

Fig. 2

Total admittance

$$Y = Y_1 + Y_2$$

$$= \frac{1}{1/j\omega C} + \frac{1}{R + j\omega L}$$

$$= j\omega C + \frac{(R - j\omega L)}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right) \quad \text{---(1)}$$

At resonance

$$\mathrm{Im}(Y) = 0 \qquad \Rightarrow \left(C - \frac{L}{R^2 + \omega^2 L^2}\right) = 0 \qquad \Rightarrow C = \frac{L}{R^2 + \omega^2 L^2}$$

$$\Rightarrow \qquad \omega^2 L^2 = \frac{L}{C} - R^2 \qquad \text{---(2)}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad \Rightarrow \qquad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R=0 $\Rightarrow$ $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ same as for series resonance