**Electrical Resonance**

Resonance is defined as the condition in a circuit containing at least one inductor and one capacitor when the supply voltage & supply current are in same phase.

At resonance

i. Angle between V & I is zero
ii. $\phi = 0 \Rightarrow \cos \phi = 1$, Unity power factor
iii. If Z and Y are given in rectangular form, their imaginary part will be zero i.e.
   - If $Z = R \pm jX \Rightarrow \text{Im}(Z) = 0 \Rightarrow X = 0$
   - If $Y = G \pm jB \Rightarrow \text{Im}(Y) = 0 \Rightarrow B = 0$
iv. If Z and Y are given in polar form, their angle will be zero i.e.
   - If $Z = |Z| \angle \theta \Rightarrow \theta = 0$
   - If $Y = |Y| \angle \theta \Rightarrow \theta = 0$

**Types of resonance:** Two types

1. **Series Resonance:** Consider the following circuit

   ![Image of series resonance circuit](image.png)

   $Z = \sqrt{R^2 + (X_L - X_C)^2}$

   If $X_L = X_C$ then
   i. Total reactance $X = X_L - X_C = 0$
   ii. Total impedance $Z = R = Z_r$ (say)
   iii. Current $I = \frac{V}{Z} = \frac{V}{R}$ maximum
   iv. Angle between V & I is zero $\Rightarrow \phi = 0 \Rightarrow \cos \phi = 1$

   This condition is called as series resonance and frequency at which it occurs is called resonance frequency ($f_r$).
So at resonance

\[ X_L = X_C \quad \Rightarrow \omega_r L = 1/\omega_r C \quad \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \quad \Rightarrow f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \]

2. **Shunt Resonance:** Consider the following parallel circuit

Total admittance

\[ Y = Y_1 + Y_2 \]

\[ = \frac{1}{1/j\omega C} + \frac{1}{R + j\omega L} \]

\[ = j\omega C + \frac{(R - j\omega L)}{R^2 + \omega^2 L^2} \]

\[ = \frac{R}{R^2 + \omega^2 L^2} + j\omega \left( C - \frac{L}{R^2 + \omega^2 L^2} \right) \quad \text{---(1)} \]

At resonance

\[ \text{Im}(Y) = 0 \quad \Rightarrow \left( C - \frac{L}{R^2 + \omega^2 L^2} \right) = 0 \quad \Rightarrow C = \frac{L}{R^2 + \omega^2 L^2} \]

\[ \Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2 \quad \text{---(2)} \]

\[ \Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \Rightarrow \quad f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \]

If \( R = 0 \) \quad \Rightarrow \quad f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \quad \text{same as for series resonance}