<u>**AC Fundamental**</u>

<u>**Simple Loop Generator:**</u> Whenever a conductor moves in a magnetic field, an emf is induced in it.

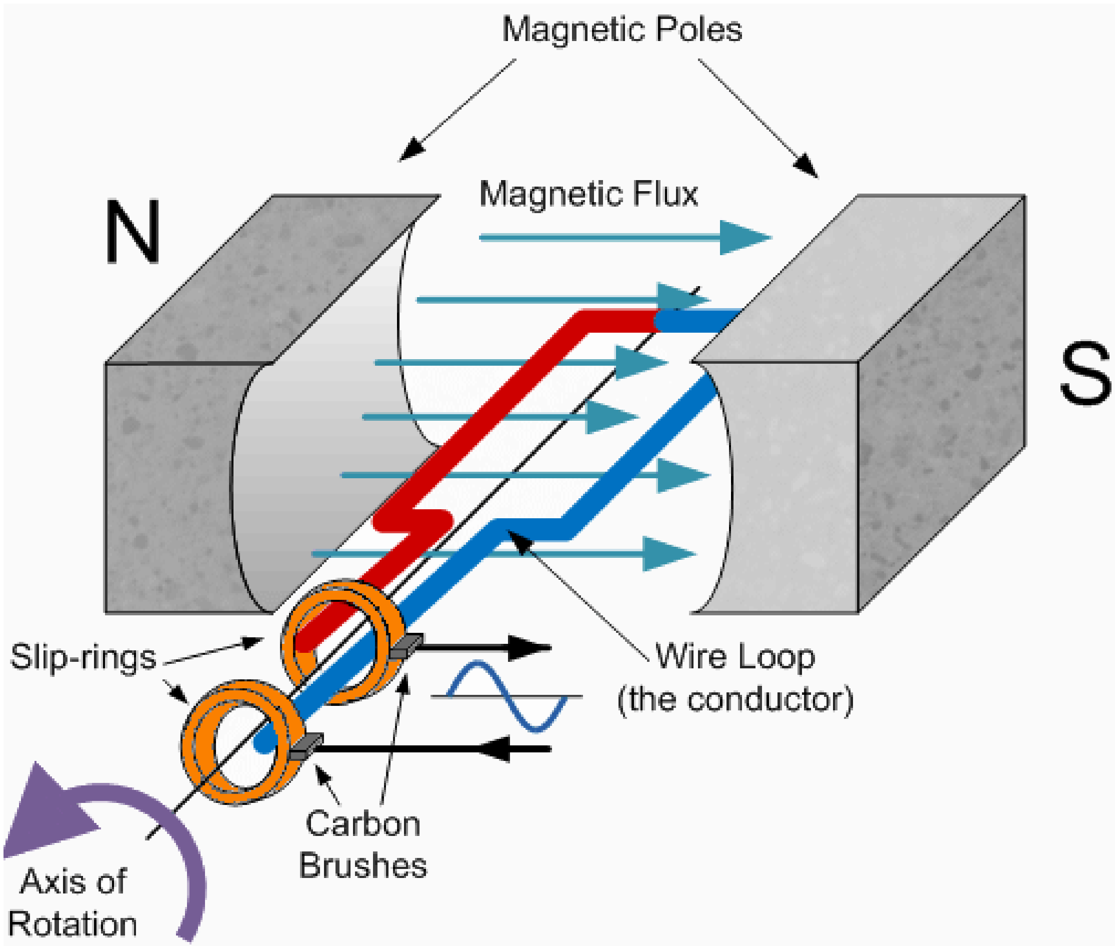


Fig.1: Simple Loop Generator

The amount of EMF induced into a coil cutting the magnetic lines of force is determined by the following three factors.

1. Speed – the speed at which the coil rotates inside the magnetic field.
2. Strength – the strength of the magnetic field.
3. Length – the length of the coil or conductor passing through the magnetic field.

What is instantaneous value?

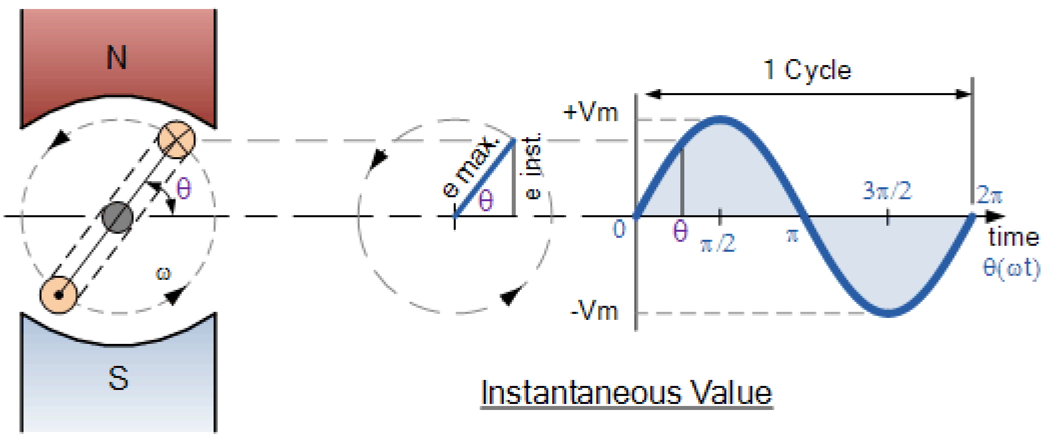


Fig.2: Instantaneous Value

The instantaneous values of a sinusoidal waveform is given as the "Instantaneous value = Maximum value x sinϴ and this is generalized by the formula.

$$v = V_m \, Sin\theta$$

Sinusoidal Waveform Construction

Coil Angle (θ)	0	45	90	135	180	225	270	315	360
e = Vmax.sinθ	0	70.71	100	70.71	0	-70.71	-100	-70.71	-0

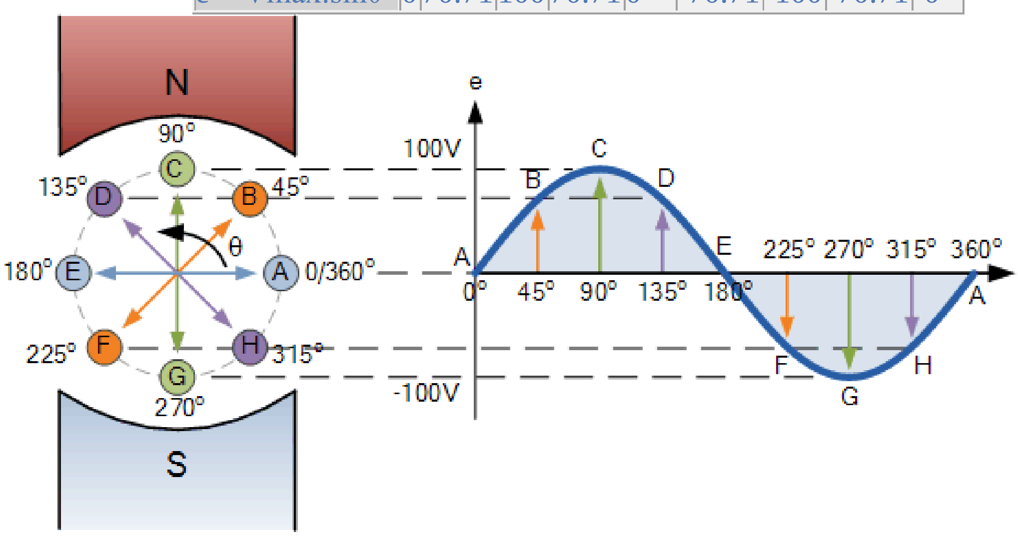


Fig.3: Wave form construction

Sinusoidal Quantities (Voltage & Current)

Voltage or EMF is denoted by

$$v = V_m \, Sin\omega t$$

OR

$$e = E_m \, Sin\omega t$$

Fig.4

Where V_m or E_m = Maximum Value of Voltage or EMF
v or e = Instantaneous value of Voltage or EMF
$\omega = 2\Pi f = Angular\ \ frequency$
f = frequency in Hz

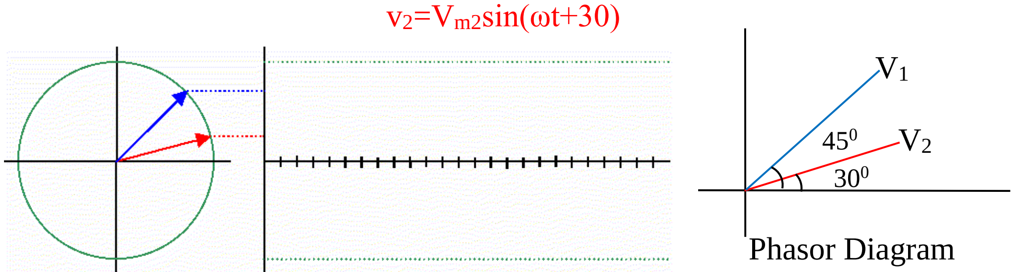
Current $i = I_m \, Sin\omega t$

Note: In AC electrical theory every power source supplies a voltage that is either a sine wave of one particular frequency or can be considered as a sum of sine waves of differing frequencies. The good thing about a sine wave such as V(t) = Asin(ωt + δ) is that it can be considered to be directly related to a vector of length A revolving in a circle with angular velocity ω. The phase constant δ is the starting angle at t = 0. Following animated GIF link shows this relation.

sine.gif

Phase difference, Phasor diagram and Leading & lagging concepts: When two sine waves are produced on the same display, they may have some phase different, one wave is often said to be *leading* or *lagging* the other. Consider following two sine waves

$v_1=V_{m1}\sin(\omega t+45)$
$v_2=V_{m2}\sin(\omega t+30)$



Phasor Diagram
(Clockwise +Ve angle)

Fig.5

Here v_1 & v_2 are having a phase difference of 15^0.The blue(v_1) vector is said to be leading the red(v_2) vector Or Conversely the red vector is lagging the blue vector.
This terminology makes sense in the revolving vector picture as shown in following GIF figure.

doublesine.gif

Displaced waveforms:

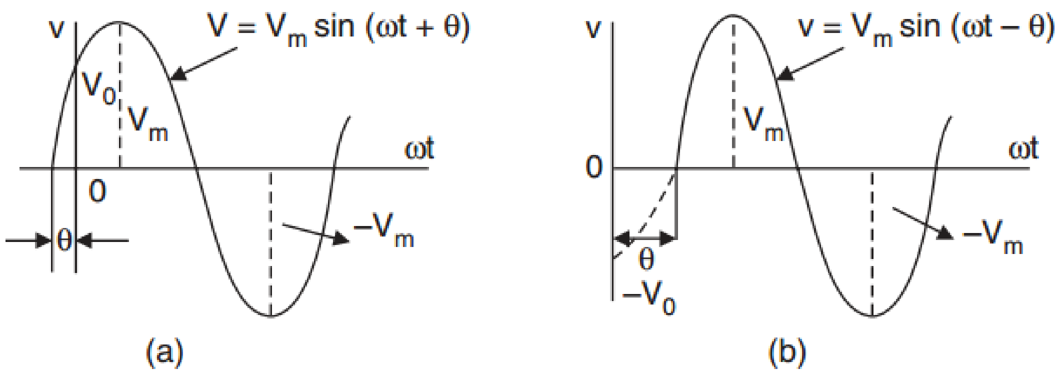


Fig. 6

Some Terminology

1. Wave form: The shape of the curve.
2. Instantaneous value: The value at any instant of time.
3. Cycle: One complete set of +ve and –ve values
4. Time Period: Time taken to complete one cycle.
5. Frequency: Number of cycles completed in one second.

$$f = \frac{1}{T}\ Hz$$

Average value of AC

Let

$$i = I_m\ Sin\omega t$$

Average current

$$I_{av} = \frac{Area\ of\ \ half\ \ cycle}{\pi}$$

Fig.7

$$= \frac{\int_0^{\pi} id\omega t}{\pi} = \frac{\int_0^{\pi} I_m Sin\omega t\ d\omega t}{\pi}$$

$$= \frac{I_m}{\pi}\int_0^{\pi} Sin\omega t\, d\omega t = \frac{I_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$I_{av} = \frac{2}{\pi} I_m$$

Similarly average value of ac voltage

$$V_{av} = \frac{2}{\pi} V_m$$

Note: Average value of the following for complete cycle = 0

$\sin\omega t$

$\cos\omega t$

$\sin(\omega t - \theta^0)$

$\cos(\omega t + \theta^0)$

RMS (Root Mean Square or Effective) Value:

Let

$$i = I_m\, Sin\omega t$$

RMS current

$$I_{rms} = I = \sqrt{\frac{Area\ of\ cycle\ of\ i^2}{\pi}}$$

$$= \sqrt{\frac{\int_0^{\pi} i^2 d\omega t}{\pi}} = \sqrt{\frac{\int_0^{\pi} I_m^2 Sin^2\omega t\, d\omega t}{\pi}}$$

$$= \sqrt{\frac{I_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d\omega t} = \sqrt{\frac{I_m^2}{\pi}\left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_0^{\pi}}$$

Fig.8

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Similarly RMS value of ac voltage

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Peak Factor(K_p):

$$K_p = \frac{Maximum\, Value}{RMS\ Value} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$$

for complete sine wave

Form Factor (K_f):

$$K_f = \frac{RMS\ Value}{\text{Avarage}\ Value} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

for complete sine wave

Example 1: Calculate the average current, effective voltage, peak factor & form factor of the output waveform of the half wave rectifier.

Solution:

Solution: Fig.9

$$V_{avg} = \frac{\int_0^{\pi} V_m Sin\omega t \, d\omega t}{2\pi} = \frac{V_m}{2\pi}[\cos 0 - \sin\pi] = \frac{V_m}{\pi}$$

$$V_{rms} = \sqrt{\frac{\int_0^{\pi} V_m^2 Sin^2\omega t \, d\omega t}{2\pi}} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_0^{2\pi}} = \frac{V_m}{2}$$

$$K_p = \frac{Maximum\,Value}{RMS\ Value} = \frac{V_m}{V_m/2} = 2$$

$$K_f = \frac{RMS\ Value}{\text{Avarage}\ Value} = \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2} = 1.57$$

Question: Find the above for the output waveform of the full wave rectifier.

Ans: $V_{avg} = \frac{2V_m}{\pi}, V_{rms} = \frac{V_m}{\sqrt{2}}, K_p = 1.414 \ \& \ K_f = 1.11$

<u>Addition and subtraction of alternating quantities:</u> Consider the following examples

Example 2: Three sinusoidal voltages acting in series are given by

v_1=10 Sin440t
v_2=10√2 Sin(440t-45)
v_3=20 Cos440t

Determine (i) an expression for resultant voltage (ii) the frequency and rms value of resultant voltage.

Solution: Phasor diagram method:

Voltage v_3 may be rewritten as

v_3=20 Sin(440t+90^0)

Phasor diagram all voltages will be as shown bellow (Taking maximum values)

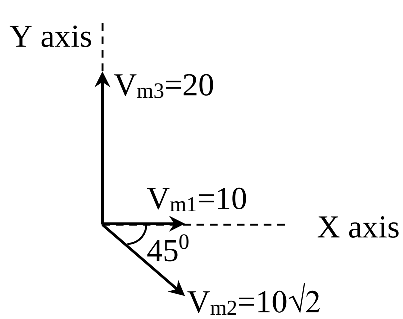


Fig. 10: Phasor Diagram

(i) X component of maximum value of resultant voltage

V_{mx}=10 Cos0^0+10√2 Cos45^0+20 Cos90^0= 20

Y component of maximum value of resultant voltage

V_{my}=10 Sin0^0 -10√2 Sin45^0 +20 Sin90^0 = 10

So maximum value of resultant voltage

$$V_m = \sqrt{V_{mx}^2 + V_{my}^2} = \sqrt{400+100} = 10\sqrt{5}$$

$$\tan\phi = \frac{V_{my}}{V_{mx}} = \frac{10}{20} \Rightarrow \phi = 26.56^0$$

So resultant voltage

v=Vm Sin(ωt+ϕ)

v=10√5 Sin(440t+26.56^0) Ans

(ii) ω=440 or 2πf=440 => **f=70 Hz** Ans

$V_{rms}=V_m/\sqrt{2}$=**15.81 V** Ans

Question: Draw the phasor diagram of following waveform

v_1=100 Sin500t

v_2=200 Sin(500t+π/3)

v_3=-50 Cos500t

v_4=150 Sin(500t-π/4)

Note: v_3=-50 Cos500t=+50 Sin(500t-π/2)

<u>Single phase AC circuit:</u>

1. <u>Purely Resistive Circuit:</u>

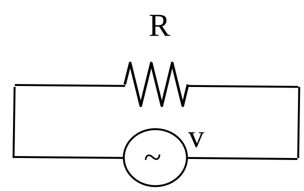


Fig. 11a

Let applied voltage

v =V_m Sinωt ----------(1)

According to Ohm's Law

v=iR

V_m Sinωt = i R

i =V_m/R Sinωt

Let $V_m/R = I_m$

So i =I_m Sinωt ----------(2)

- **Phasor diagram**

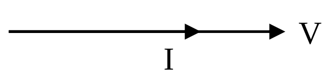


Fig .11b

Φ =0, PF cos Φ=1, No phase different or V & I are in same phase

> **Note:**
> 1. Cosine of angle between Voltage (V) and current (I) is known as power factor i.e PF= Cosϕ
> 2. If current lags => power factor lags
> 3. If current leads => power factor leads

- **Power consumed**

  Instantaneous power

  $p=v*i$

  $= V_m \sin\omega t * I_m \sin\omega t$

  $= V_m I_m \sin^2\omega t$

  Average power

$$P = \frac{\int_0^{\pi} p\,d\omega t}{\pi} = \frac{1}{\pi}\int_0^{\pi} V_m I_m Sin^2\omega t\, d\omega t$$

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t = \frac{1}{\pi}\int_0^{\pi} \frac{V_m I_m}{2}\, d\omega t - \frac{1}{\pi}\int_0^{\pi} \cos 2\omega t$$

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{rms} I_{rms} = VI$$

- **Waveform**

Fig. 11c

Note: It is also clear from the above graph that p is having some average value.

Example 3: A 250 Volts (RMS), 50 Hz voltage is applied across a circuit consisting of a pure resistance of 20 ohm. Determine

i. The current flowing through the circuit
ii. Power absorbed by circuit
iii. Expression for voltage and current
iv. Draw waveform and phasor diagram.

Solution:

i. $I_{rms} = I = \frac{V}{R} = \frac{250}{20} = 12.5 Amp$ Ans

ii. Power absorbed =VI=250*12.5=3125 Watt Ans

iii. $v = 250\sqrt{2}\sin 2\pi(50)t = 250\sqrt{2}\sin 100\pi t$ Ans

$i = 12.5\sin 100\pi t$ Ans

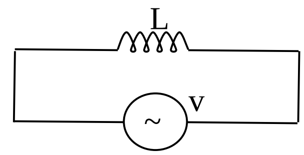
iv. See figure 11c

2. Purely Inductive Circuit:

Let applied voltage

$v = V_m \, Sin\omega t$ ---------- (1)

Fig. 12a



When an alternating voltage is applied to purely inductive coil, an emf known as self induced emf is induced in the coil which opposes the applied voltage.

Self induced emf

$$e_L = -L\frac{di(t)}{dt}$$

Since applied voltage at every instant is equal and opposite to self induced emf

We have $v = -e_L$

$$V_m Sin\omega t = -\left[-L\frac{di(t)}{dt}\right]$$

$$di = \frac{1}{L}V_m Sin\omega t \, dt$$

Integrating

$$i = \int \frac{1}{L}V_m Sin\omega t \, dt = \frac{1}{L}V_m\left[\frac{Cos\omega t}{\omega}\right] = \frac{V_m}{\omega L}Sin(\omega t - \pi/2)$$

Let $I_m = \frac{V_m}{\omega L}$

Here $\omega L = X_L$ know as inductive reactance

So $i = I_m Sin(\omega t - \pi/2)$

Here current lags by 90^0.

- **Phasor diagram**

  $\Phi = 90^0$, PF $Cos\Phi = 0$, Phase different between V & I is 90^0. Current (I) lags by 90^0 from voltage (V).

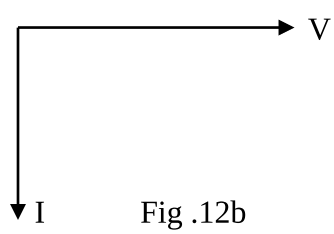


Fig .12b

- **Power consumed**

  Instantaneous power

  $p = v*i$

  $= V_m Sin\omega t * I_m Sin(\omega t - 90^0)$

  $= -V_m I_m Sin\omega t \, Cos\omega t$

  $= -V_m I_m /2 \, Sin2\omega t$

  Average power

$$P = \frac{\int_0^\pi p d\omega t}{\pi} = Ava\left[-\frac{V_m I_m}{2}Sin2\omega t\right] = 0$$

Hence NO power is consumed in purely inductive circuit.

- **Waveform**

Fig. 12c

Note: It is also clear from the above graph that p is having ZERO average value.

Example 4: The voltage and current thorough a circuit element are $v = 100\sin(314t + 45^0)V$ and $i = 10\sin(314t + 315^0)A$.

i. Identify the circuit elements
ii. Find the value
iii. Obtain expression for power

Solution:

i. Phase difference = 315-45 =90^0 and I lags v by 90^0, see the phasor diagram.
So it is pure inductor

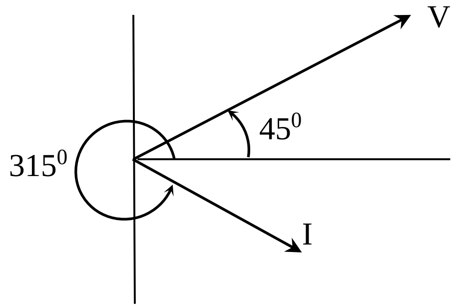


Fig .13

ii. Inductance X_L=ωL => 10=314L => L=10/314 H Ans

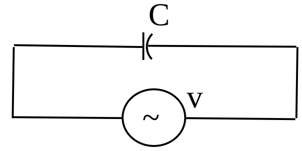
iii. $p = 100\sin(314t + 45^0) \times 10\sin(314t + 315^0) = \dfrac{1000}{2}\sin 628t$ Ans

3. Purely Capacitive Circuit:

Let applied voltage

$$v = V_m \,\mathrm{Sin}\omega t \text{ ---------- } (1)$$

Fig. 14a



Charge $\quad Q = Cv$

$$Q = CV_m Sin\omega t$$

And we know $\quad i = \frac{dq}{dt} = \frac{d}{dt} CV_m Sin\omega t\, dt = V_m C\omega\, Cos\omega t$

$$i = \frac{V_m}{1/\omega C} Sin(\omega t + \pi/2)$$

Let $\quad I_m = \frac{V_m}{1/\omega C}$

Here $\quad \frac{1}{\omega C} = X_C$ know as capacitive reactance

So $\quad i = I_m Sin(\omega t + \pi/2)$

Here current leads by 90^0.

- **Phasor diagram**

  $\Phi = 90^0$, PF $Cos\Phi = 0$,
  Phase different between V & I is 90^0.
  Current (I) leads by 90^0 from voltage (V).

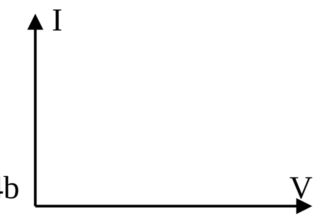


Fig .14b

- **Power consumed**

  Instantaneous power

  $p = v * i$

  $= V_m \, Sin\omega t * I_m \, Sin(\omega t + 90^0)$

  $= V_m I_m \, Sin\omega t \, Cos\omega t$

  $= V_m I_m /2 \, Sin2\omega t$

  Average power

$$P = \frac{\int_0^{\pi} p\, d\omega t}{\pi} = Ava\left[\frac{V_m I_m}{2} Sin2\omega t\right] = 0$$

  Hence NO power is consumed in purely capacitive circuit.

- **Waveform**

Fig. 14c

Note: It is also clear from the above graph that p is having ZERO average value.

Example 5: A 318 µF capacitor is connected to 200 V, 50 Hz supply. Determine

i. Capacitive reactance offered by the capacitor
ii. The maximum current
iii. RMS value of current drawn by capacitor
iv. Expression for voltage & current

Solution:

i. $X_C=1/\omega C=1/2\pi fC=10$ Ohm Ans
ii. $I_m=V_m/X_C=200\sqrt{2}/10=20\sqrt{2}$ A Ans
iii. $I_{rms} = I_m/\sqrt{2}=20$ A Ans
iv. $v = 200\sqrt{2}\sin 100\pi t$ and $i = 20\sqrt{2}\sin(100\pi t + \pi/2)$ Ans

4. R-L Circuit:

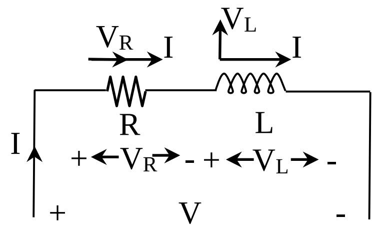


Fig. 15a

Let the applied voltage is

$$v = V_m \sin\omega t \text{ ---------- (1)}$$

Voltage across resistance $V_R=I.R$

Voltage across inductance $V_L=I.X_L$

$=I.(\omega L) = I.(2\pi fL)$

From phasor diagram

$$V=\sqrt{V_R{}^2+V_L{}^2}=\sqrt{(IR)^2+(X_L I)^2}$$

$$V = I\sqrt{R^2+X_L{}^2} = IZ$$

Where $Z=\sqrt{R^2+X_L{}^2}$ know as impedance

From phasor diagram, "I" is lagging behind "V" by an angle ϕ

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \Rightarrow \phi = \tan^{-1}\frac{X_L}{R}$$

So $i = I_m Sin(\omega t - \phi)$

Where $I_m = \dfrac{V_m}{Z}$

- **Phasor diagram**

Phase different between V & I is ϕ. Current (I) lags by ϕ^0 from voltage (V).

V_R & I are in same phase, V_L is leading by an angle of 90^0 from I etc.

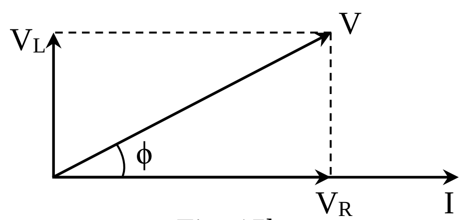


Fig .15b

- **Power consumed**

Instantaneous power

$$p = v * i$$
$$= V_m \sin\omega t * I_m \sin(\omega t - \phi^0)$$
$$= \frac{V_m I_m}{2} 2 \sin\omega t \sin(\omega t - \phi) = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

Average power

$$P = Ava\left[\frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]\right]$$
$$= Ava\left[\frac{V_m I_m}{2}\cos\phi\right] - Ava\left[\frac{V_m I_m}{2}\cos(2\omega t - \phi)\right]$$
$$P = \frac{V_m I_m}{2}\cos\phi \qquad \left[\because Ava\cos(2\omega t - \phi) = 0\right]$$
$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi$$

- **Waveform**

Fig. 15c

- **Voltage triangle and Impedance triangle:**

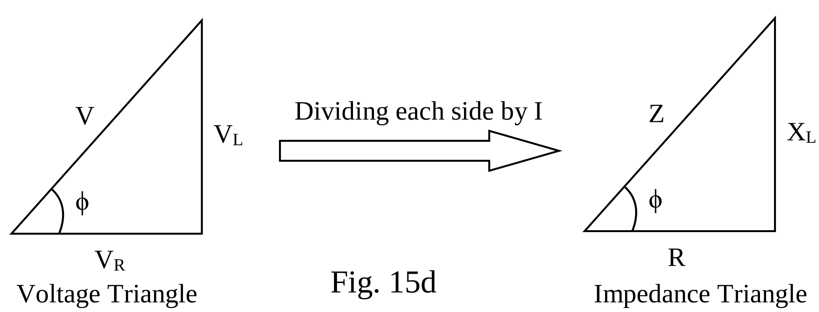
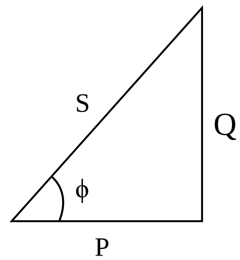


Fig. 15d

Power factor: $\cos\phi = V_R/V = R/Z$

- **Active power, reactive power, apparent power and power factor:**

Active power	$P = VI \cos\phi = I^2R$	Unit: Watt, KW
Reactive power	$Q = VI \sin\phi = I^2X$	Unit: VAR, KVAR
Apparent Power	$S = VI = I^2Z$	Unit: VA, KVA



Power Triangle Fig. 15e

Power factor: cos ϕ= Active power/Apparent power

Example 6: A resistance and inductance are connected in series across a voltage $v = 283\sin(314t)$ & current expression as $i = 4\sin(314t - \pi/4)$. Find the value of resistance, inductance and power factor.

Solution:

$Z=V_m/I_m=70.75$

$R=Z \cos\phi=70.75^* \cos(\pi/4)=50.027$ ohm Ans

$X_L=Z \sin\phi=70.75^* \sin(\pi/4)=50.027$ ohm $=\omega L \Rightarrow L = 0.1592$ H Ans

Power factor = $\cos\phi= \cos(\pi/4)=1/\sqrt{2}=0.707$ Ans

Example 7: $i = 14.14\sin(\omega t - \pi/6)$ passes in an electric circuit when a voltage of $v = 141.4\sin\omega t$ is applied to it. Determine the power factor of the circuit, the value of true power, apparent power and circuit components.

Solution:

$Z=V_m/I_m=141.4/14.14=10$ Ohm

PF $\cos\phi=\cos(\pi/6)=0.866$ Ans

$P=VI \cos\phi =(141.4/\sqrt{2})(14.14/\sqrt{2})*0.866=866$ W Ans

$S=VI=(141.4/\sqrt{2})(14.14/\sqrt{2})=1000$ W Ans

$R=Z\cos\phi=10*0.866=8.66$ Ans

$X_L=Z\sin\phi=10*0.5=5$ Ohm Ans

5. **R-C Circuit:**

Let the applied voltage is

$$v = V_m \sin\omega t \text{ ---------- (1)}$$

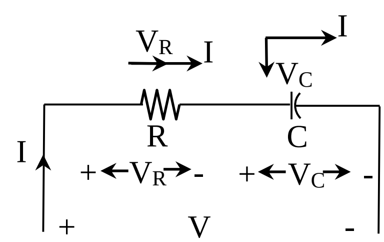


Fig. 16a

Voltage across resistance $V_R=I.R$

Voltage across inductance $V_C=I.X_C$

$=I.(1/\omega C) = I. (1/2\pi fC)$

From phasor diagram

$$V = \sqrt{V_R{}^2 + V_C{}^2} = \sqrt{(IR)^2 + (X_C I)^2}$$

$$V = I\sqrt{R^2 + X_C{}^2} = IZ$$

Where $Z = \sqrt{R^2 + X_C{}^2}$ know as impedance

From phasor diagram, "I" is leading "V" by an angle ϕ

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \Rightarrow \phi = \tan^{-1}\frac{X_C}{R}$$

So $i = I_m Sin(\omega t + \phi)$

Where $I_m = \dfrac{V_m}{Z}$

- **Phasor diagram**

  Phase different between V & I is ϕ. Current (I) leads by ϕ^0 from voltage (V).

  V_R & I are in same phase, V_C is lagging by an angle of 90^0 from I etc.

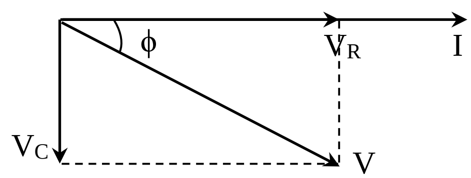


Fig .16b

- **Power consumed**

  Instantaneous power

  p=v*i

  $= V_m \sin\omega t * I_m \sin(\omega t + \phi^0)$

  $$= \frac{V_m I_m}{2} 2\sin\omega t Sin(\omega t + \phi) = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$$

  Similar to R-L circuit average power

  $$P = VI\cos\phi$$

- **Waveform**

Fig. 16c

Note: Similar to R-L circuit we can have voltage triangle and power triangle.

6. **R-L-C Circuit:**

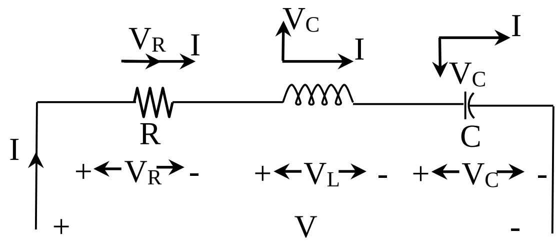


Fig. 17a

Let the applied voltage is

$$v = V_m \, Sin\omega t \text{ ---------- } (1)$$

Voltage across resistance $V_R = I.R$

Voltage across inductance $V_L = I.X_L$

Voltage across inductance $V_C = I.X_C$

From phasor diagram

$$V = \sqrt{V_R^{\,2} + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} = IZ$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ know as impedance

From phasor diagram, "I" is lagging "V" by an angle ϕ

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R} \Rightarrow \phi = \tan^{-1}\frac{X_L - X_C}{R}$$

So $i = I_m Sin(\omega t - \phi)$

Where $I_m = \dfrac{V_m}{Z}$

- **Phasor diagram**

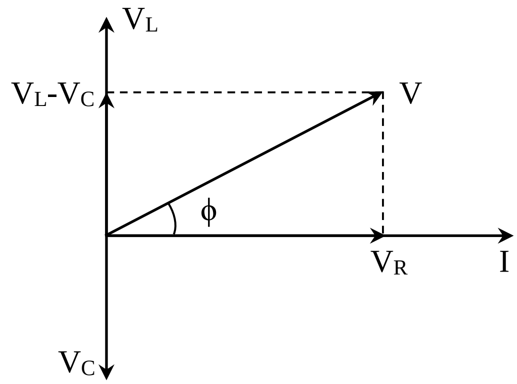


Fig .17b Assuming $V_L > V_C$

- **Power consumed**
  Average power (similar to R-L circuit)
  $P = VI\cos\phi$
- **Different cases:**
  1. VL>VC => Inductive circuit, "I" lags "V"
  2. VL<VC => Capacitive circuit, "I" leads "V"
  3. VL=VC => Resistive circuit, "I" & "V" are in same phase, power factor PF is Unity. This condition is called as electrical resonance.

Example 8: A series RLC circuit consisting of resistance of 20 Ohm, inductance of 0.2 H and capacitance of 150 µF is connected across a 230 V, 50 Hz source. Calculate

I. Current
II. Magnitude & nature of power factor

Solution:

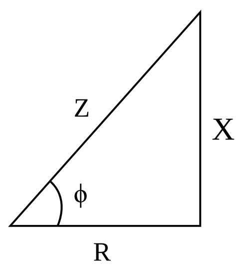
R=20 Ohm
X_L=2лfL=2л*50*0.2=62.8 Ohm
X_C=1/2лfC=1/(2л*50*150*10^{-6})=21.23

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{20^2 + (62.8 - 21.23)^2} = 46.13$$

i. I=V/Z=4.98 A Ans
ii. PF=R/Z=0.433 lagging (As X_L>X_C) Ans

Impedance(Z):

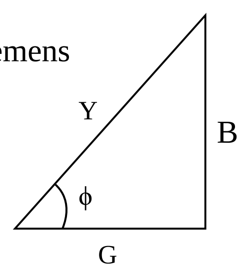
$Z = \sqrt{R^2 + X^2}$ Ω, Ohm
Where R= Resistance Ω
X= Reactance Ω
PF: Cosϕ= R/Z



Impedance Triangle Fig. 18

Admittance(Y): It is reciprocal of impedance

Y=1/Z
$Y = \sqrt{G^2 + B^2}$ Ω^{-1}, Mho, Siemens
Where G= Conductance Ω^{-1}
X= Susceptance Ω^{-1}
PF: Cosϕ= G/Y
G=YCos ϕ=(1/Z)(R/Z)=R/Z^2.
B=YSin ϕ=(1/Z)(X/Z)=X/Z^2.



AdmittanceTriangle Fig. 19

<u>Parallel Circuits: There are 3 methods for solving parallel circuits</u>

1. **<u>Phasor Method:</u>** Consider the following circuit

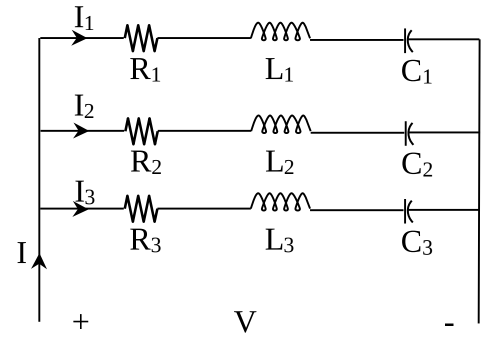


Fig. 20a

For branch 1 $Z_1 = \sqrt{R_1^{\,2} + (X_{L1} - X_{C1})^2}$

$$I_1 = \frac{V}{Z_1}$$

$$\cos\Phi_1 = \frac{R_1}{Z_1}$$

$+\Phi_1$ (Leading PF) if $X_{L1} > X_{C1}$ (Inductive circuit)

$-\Phi_1$ (Lagging PF) if $X_{L1} < X_{C1}$ (Capacitive circuit)

Similarly for branch 2 and 3 we have Z_2, I_2, $\cos\phi_2$ and Z_3, I_3, $\cos\phi_3$ respectively.

Now we can draw the phasor diagram as shown bellow assuming any values for I_1, I_2, I_3 and ϕ_1, ϕ_2, ϕ_3

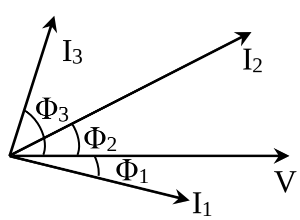


Fig .20b

Resultant current I which will be the phasor sum of I_1, I_2 & I_3 can be determine by using any of following method

i. Parallelogram method
ii. Resolving branch current I_1, I_2 & I_3 along x and y-axis.

Component of resultant current "I" along x-axis

$$I\cos\Phi = I_1\cos\Phi_1 + I_2\cos\Phi_2 + I_3\cos\Phi_3$$

Component of resultant current "I" along x-axis

$$I\sin\Phi = I_1\sin\Phi_1 + I_2\sin\Phi_2 + I_3\sin\Phi_3$$

$$I=\sqrt{(ISin\Phi)^2+(ICos\Phi)^2} \quad \& \quad \tan\Phi=\frac{ISin\Phi}{ICos\Phi}$$

2. **Admittance Method:** Consider the following circuit

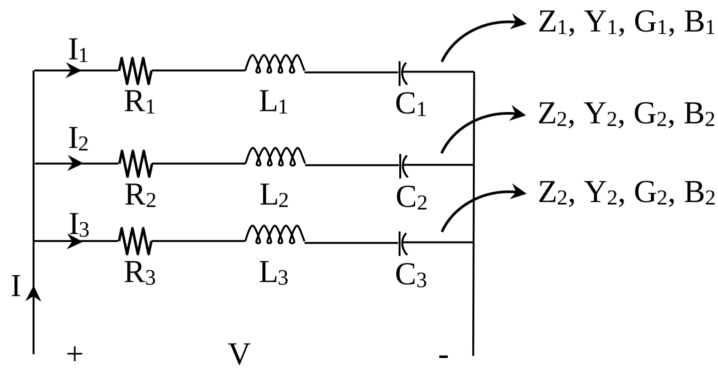


Fig. 21

We know conductance $G=\frac{R}{Z^2}$ => find out G_1, G_2 & G_3

We know susceptance $B=\frac{X}{Z^2}$ => find out B_1, B_2 & B_3

Total conductance

$G=G_1+G_2+G_3$

Total susceptance

$B=B_1+B_2+B_3$

Total Admittance

$Y=\sqrt{G^2+Y^2}$

So $V=IZ$ => $I=V/Z=VY$

$I_1=VY_1$

$I_2=VY_2$

$I_3=VY_3$

3. **Symbolic or j-notation Method:**

Consider following voltage and current

$v=V_m \sin\omega t$

$i=I_m \sin(\omega t-\theta)$

Wave form and phasor diagram will be as shown bellow

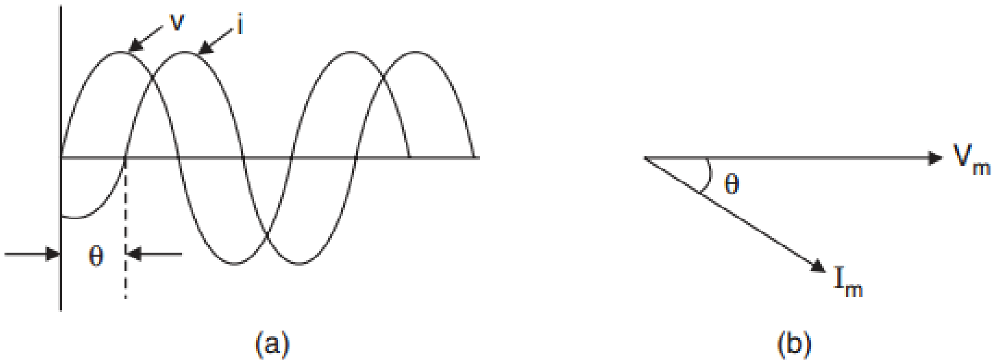


Fig. 22a

Therefore either the time waveform of the rotating phasor or the phasor diagram can be used to describe the system. Since both the diagrams, the time diagram and the phasor diagram convey the same information, the phasor diagram being much more simpler, it is used for an explanation in circuit theory analysis. Since electrical data is given in terms of rms value, we draw phasor diagram with phasor values as rms rather than peak value used so far.

Above voltage can be represented by

$V = V_x + jV_y$ Cartesian Co-ordinates

$V = |V| \angle\theta$ in polar coordinates.

Here

V_x=Component of V along x-axis

V_y= Component of V along y-axis

j is an operator which when multiplied to a phasor rotates the phasor 90° anticlockwise and $j = \sqrt{(-1)}$ or $j^2 = -1$

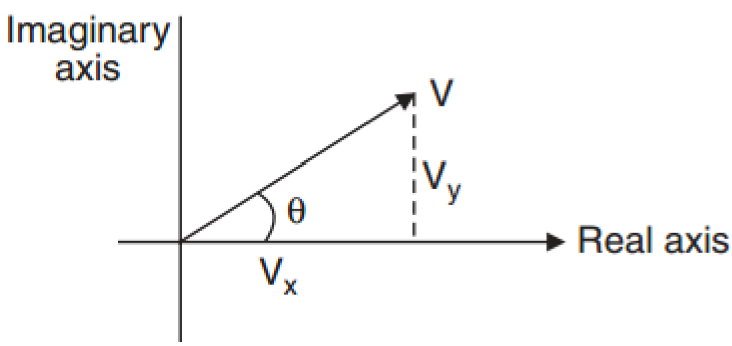


Fig. 22b: Phasor Representation

Hence a voltage or current can be represented by a complex number.

Note:

In Phasor algebra:

- Addition & Subtraction is done in Cartesian form.
- Multiplication, Division, power and roots are done in polar form.

Phasor Algebra applied to single phase circuit

I. R-L series circuit:

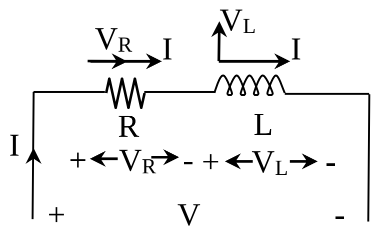


Fig. 23

- Consider I as reference $I = I\angle 0^0$
  $= I + j0$

  Voltage drop across resistance V_R=IR+j0

  Voltage drop across inductance $V_L=0+jX_L$

  So total voltage $V=V_R+V_L$

  $=(IR+j0)+(0+jX_L)$

  $=IR+jX_L =I(R+jX_L)$

  V=IZ

  Where $Z=R+jX_L= Z\angle\theta$

- Consider V as reference $V=V+j0=V\angle 0^0$

  $$I = \frac{V}{Z} = \frac{V\angle 0^0}{Z\angle\theta} = \frac{V}{Z}\angle -\theta$$

II. R-C series circuit:

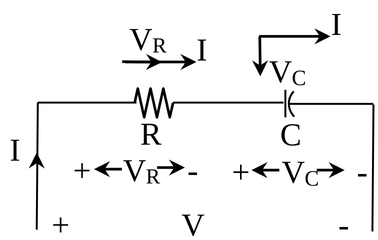


Fig. 24

- Consider I as reference $I = I\angle 0^0$
  $= I + j0$

  Voltage drop across resistance V_R=IR+j0

  Voltage drop across capatance $V_L=0-jX_C$

  So total voltage $V=V_R+V_C$

  $=(IR+j0)+(0-jX_C)$

  $=IR-jX_C =I(R-jX_C)$

  V=IZ

  Where $Z=R-jX_C= Z\angle -\theta$

- Consider V as reference $V=V+j0=V\angle 0^0$

$$I = \frac{V}{Z} = \frac{V\angle 0^0}{Z\angle -\theta} = \frac{V}{Z}\angle\theta$$

- Power determination:

  Let v=2+3j volts & i=3-j1 Amp

  $v \times \bar{i} = (2+3j)\times(3+j1) = 3+j11$

  Here

  Active power or real power $= \mathrm{Re}(v\times\bar{i})$=3 Watt

  Reactive power $= \mathrm{Im}(v\times\bar{i})$=11 VAR

III. R-L-C parallel circuit:

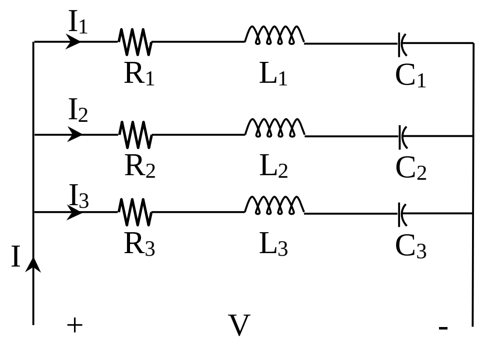


Fig. 25

For branch 1:

$Z_1=R_1+jX_1$ $\quad Where\ X_1 = X_{L1} - X_{C1}$

$$I_1 = \frac{V}{Z_1} = \frac{V+j0}{R_1+jX_1} = \frac{V+j0}{R_1+jX_1}\times\left(\frac{R_1-jX_1}{R_1-jX_1}\right)$$

$$I_1 = V\left(\frac{R_1}{R_1^2+X_1^2} - j\frac{X_{L1}}{R_1^2+X_1^2}\right)$$

Similarly for branch 2 & 3

$$I_2 = V\left(\frac{R_2}{R_2^2+X_2^2} - j\frac{X_2}{R_2^2+X_2^2}\right) \quad Where\ X_2 = X_{L2} - X_{C2}$$

$$I_3 = V\left(\frac{R_3}{R_3^2+X_3^2} - j\frac{X_3}{R_3^2+X_3^2}\right) \quad Where\ X_3 = X_{L3} - X_{C3}$$

So total current

$I=I_1+I_2+I_3$

IV. Series-parallel circuit:

- **Impedances in series**

  $Z_{eq}=Z_1+Z_2+Z_3+\ldots.$

- **Impedances in parallel**

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots$$

- A series –parallel ac circuit can be solved in same manner as that of DC series parallel circuit except that complex impedances are used instead of resistances.

Example 9: A 15mH inductor is in series with a parallel combination of a 20 ohm resistance & 20 μF capacitor. If "ω" of applied voltage is 1000. Find

i. Total impedance
ii. Total admittance
iii. Current in each branch if applied voltage is 230 V

Solution:

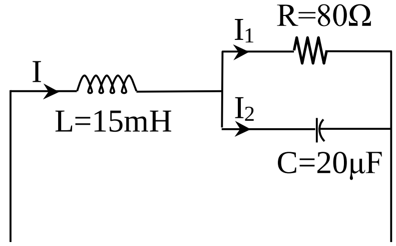


Fig. 26

Z_1=R=80 Ω

$Z_2=-jX_C=-j\frac{1}{\omega C}=\frac{-j}{1000\times 20\times 10^{-6}}=-50j\ \Omega$

$Z_3=jX_L=j\omega L=j1000\times 15\times 10^{-3}\ \Omega \qquad =15j\Omega$

Equivalent impedance

$$Z=\left(Z_1\|Z_2\right)+Z_3=\frac{Z_1Z_2}{Z_1+Z_2}+Z_3=\frac{80\times -50j}{80-50j}+15j$$

$$Z = 22.4719 - 20.9551j = 30.726\angle -43^0$$

i. $Y=\frac{1}{Z}=\frac{1}{30.726\angle -43^0}=0.0325\angle 43^0=0.0238+0.0222j$ Ans

ii. $I=\frac{V}{Z}=\frac{230\angle 0^0}{30.726\angle -43^0}=7.4854\angle -43^0=4.1552+6.2262j$ Ans

iii. $I_1=\frac{Z_2}{Z_1+Z_2}I$ By current division rule

$=\frac{-50j}{80-50j}\times 7.4854\angle -43^0=3.9655-0.1186j=3.963\angle -1.7131^0$ Ans

$I_1=\frac{Z_1}{Z_1+Z_2}I$ By current division rule

OR

$I_2=I-I_1=(4.1552+6.2262j)-(3.9655-0.1186j)=0.1897+6.3448j$ Ans