DEHRADUN INSTITUTE OF TECHNOLOGY
DEPARTMENT OF ELECTRICAL ENGINEERING
Assignment:-3

Modern Control System **State Space Analysis**

1. A system is described by the following transfer function $G(s) = \frac{20(10s+1)}{s^3+3s^2+2s+1}$ .Find the state and output equation of the system

2. Write the state equation for the circuit shown in figure 1. Take V_1 & V_2 as two inputs.

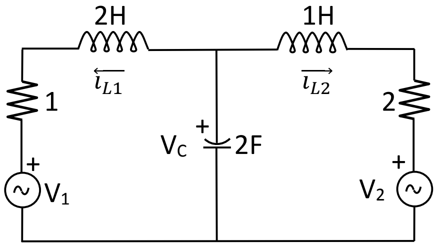

Figure 1

3. Construct the state model of a system characterized by the differential equation
$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + \frac{dy}{dt} + 6y = u$$

4. The system equations are given by
$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$
Find the transfer function of the system.

5. Find out model matrix and diagonal matrix for the systems whose system matrix are given below:
   a. $A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}$ b. $A = \begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix}$

6. Compute STM for $A = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$ by series summation method as well as by Laplace transformation method.

7. Find the time response of the system described by the equation
$$\dot{x}(t) = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$x(0) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad u(t) = 1, t > 0$$

8. Test for controllability and observability for the following systems
   a. $\dot{x}(t) = \begin{bmatrix} -0.5 & 0 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \; and \quad y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$
   b. $\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t) \; and \quad y(t) = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} x(t)$

Answers

1. $\dot{x}\,(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} u(t) \qquad y(t) = [1 \quad 10 \quad 0]x(t)$

2. $\begin{bmatrix} \dot{\iota_{L1}}\,(t) \\ \dot{\iota_{L2}}\,(t) \\ \dot{V_C}\,(t) \end{bmatrix} = \begin{bmatrix} -1/2 & 0 & 1/2 \\ 0 & -2 & 1 \\ -1/2 & -1/2 & 0 \end{bmatrix} \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \\ V_C(t) \end{bmatrix} + \begin{bmatrix} -/12 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

3. $\dot{x}\,(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \qquad y(t) = [1 \quad 0 \quad 0]x(t)$

4. $G(s) = \frac{1}{s^2+3s+2}$

5. $a. \quad M = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -4 & -3 \\ -1 & 1 & 3 \end{bmatrix} \&\ \wedge = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

   $b. \quad M = \begin{bmatrix} 0 & 2 & 3 \\ 8 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \&\ \wedge = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

6. $\begin{bmatrix} e^{-t} & e^{-t} - e^{-2t} \\ 0 & e^{-2t} \end{bmatrix}$

7. $\frac{1}{2} - 2e^{-t} + \frac{1}{2}2^{-2t}$

8. a. Uncontrollable & Unobservable b. Uncontrollable & Unobservable