1. State the Lyapunov stability theorem.
2. State the Lyapunov asymptotically stability theorem.
3. State the Lyapunov globally asymptotically stability theorem.
4. What is Lyapunov instability theorem?
5. Consider the system described by

 $\dot{x}\_{1}=x\_{2-}x\_{1}(x\_{1}^{2}+x\_{2}^{2})$ $\dot{x}\_{2}=-x\_{1-}x\_{2}(x\_{1}^{2}+x\_{2}^{2})$

Determine the equilibrium point(s) and find the stability using Lyapunov stability criterion.

Hint: take $V\left(X\right)=x\_{1}^{2}+x\_{2}^{2}$ show it pdf and then show $\dot{V}(X)$ ndf or nsdf,

Answer: (0, 0), origin is globally asymptotically stable.

1. Consider the second order system described by$\left[\begin{matrix}\dot{x}\_{1}\\\dot{x}\_{2}\end{matrix}\right]=\left[\begin{matrix}0&1\\-1&-1\end{matrix}\right]\left[\begin{matrix}x\_{1}\\x\_{2}\end{matrix}\right]$. Determine the equilibrium point(s) and find the stability using Lyapunov stability creation for LTI system.

Answer: (0, 0), if Q = I then $P=\left[\begin{matrix}\frac{3}{2}&\frac{1}{2}\\\frac{1}{2}&1\end{matrix}\right]$ and P is pd matrix, Origin is globally asymptotically stable

1. What are the different methods to find out Lyapunov functions? Explain Krasovskii’s method.
2. Consider the second order system described by$\left[\begin{matrix}\dot{x}\_{1}\\\dot{x}\_{2}\end{matrix}\right]=\left[\begin{matrix}0&1\\-1&-1\end{matrix}\right]\left[\begin{matrix}x\_{1}\\x\_{2}\end{matrix}\right]$. Determine the equilibrium point(s) and find the stability using Lyapunov stability creation use Krasovskii’s method to select the Lyapunov function.
3. Explain the Popov’s criterion to find the stability of a nonlinear system.
4. Consider the system defined by $\dot{X}=AX+Bu$ where $A=\left[\begin{matrix}0&1\\20.6&0\end{matrix}\right]$ $B=\left[\begin{matrix}0\\1\end{matrix}\right]$

By using the state feedback control u=-KX, it is desired to have closed loop poles at $s=-1.8\pm j2.4$. Determine the sate feedback gain matrix K by all three methods.

Answer: $K=\left[\begin{matrix}29.6&3.6\end{matrix}\right]$

1. Consider the system defined by $\dot{X}=AX+Bu$ $Y=CX$where

$A=\left[\begin{matrix}0&1&0\\0&0&1\\-6&-11&-6\end{matrix}\right]$ $B=\left[\begin{matrix}0\\0\\1\end{matrix}\right]$ $C=\left[\begin{matrix}1&0&0\end{matrix}\right]$

Design a full order state observer by all three methods, it is desired to have closed loop poles at $s=-2\pm j3.464$ and s=-5.

Answer: $K\_{e}=\left[\begin{matrix}3&7&-1\end{matrix}\right]^{T}$