Three Phase System

Generation of three phase system: Consider following 3 phase simple loop generator shown in figure 1a. Its generated voltage will be as shown in figure 1b.

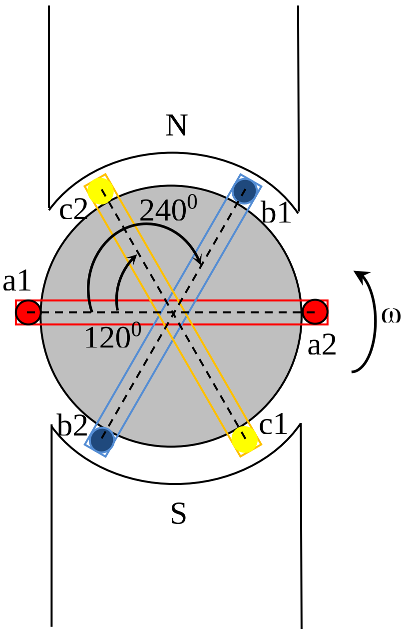


Fig.1a

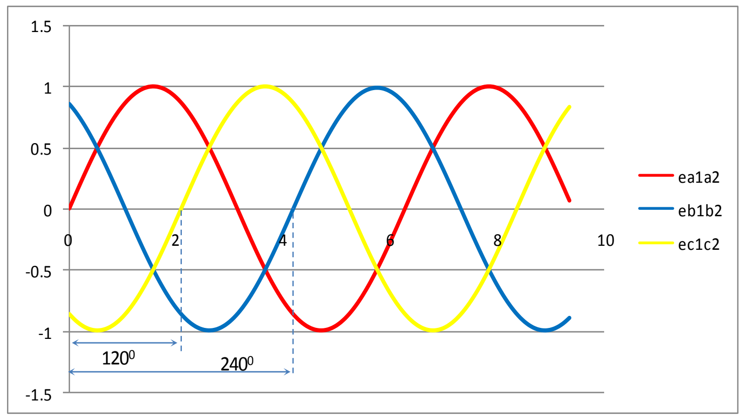


Fig.1b: 3-Phase wave from

$$e_{a1a2} = E_m Sin(\omega t) \qquad \text{-----(1)}$$

$$e_{c1c2} = E_m Sin(\omega t - 120^0) \qquad \text{-----(2)}$$

$$e_{b1b2} = E_m Sin(\omega t - 240^0) \qquad \text{-----(3)}$$

E_{a1a2}, E_{b1b2} & E_{c1c2} are RMS values.

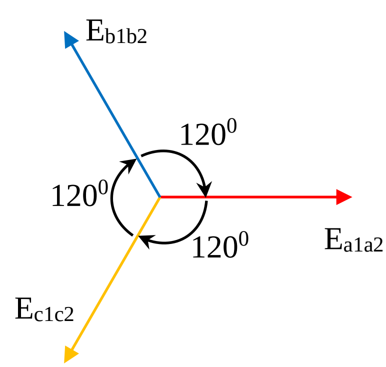


Fig.1c: Phasor Diagram

Double Subscript notation:

$-V_{AB}=+V_{BA}$

$I_{BA}=-I_{AB}$

$V_{AB}=V_A-V_B$

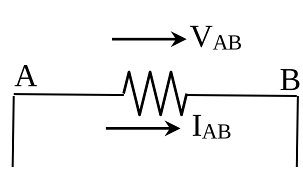


Fig. 2

Phase Sequence: It is the sequence in which current or voltages in different phases attain their maximum values.

Phase sequence may be R-Y-B or R-B-Y as shown bellow

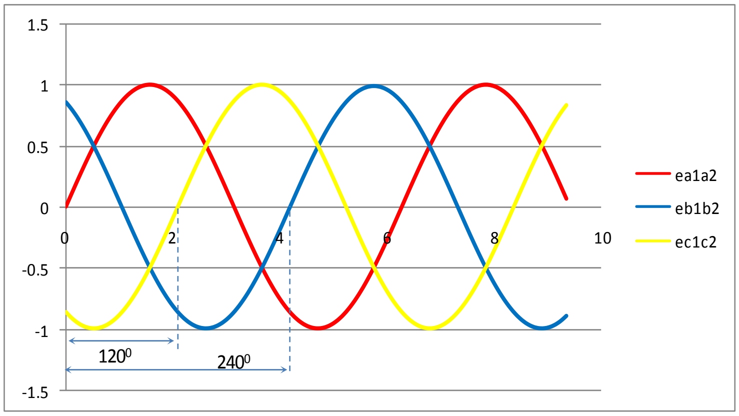


Fig.2a: Phase Sequence R-Y-B

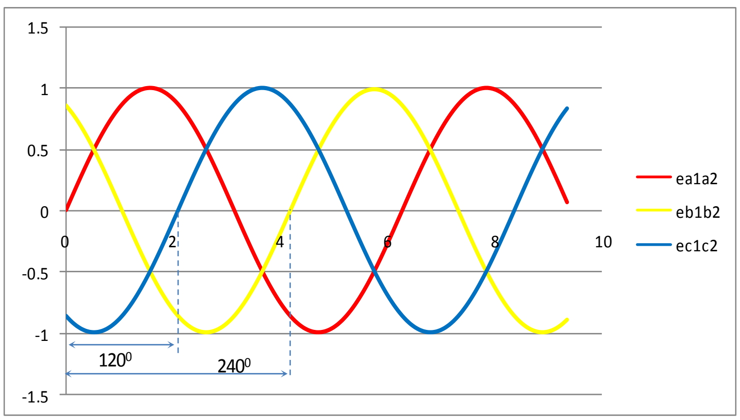


Fig.2b: Phase Sequence R-B-Y

Interconnection of 3-phase system: The six terminals of three phase winding can be connected to form any of the below.

1. Star or WYE (Y) connected 3-ϕ System
2. Mesh or Delta(Δ) connected 3- ϕ System

Star or WYE (Y) connected 3-ϕ System:

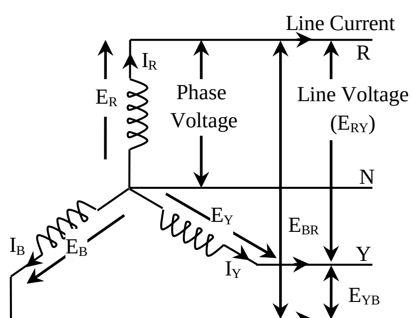


Fig.3a

E_R, E_Y & E_B are called phase voltages.

I_R, I_Y & I_B are called phase currents.

E_{RY}, E_{YB} & E_{BR} are called line voltages.

From figure it is clear that

$I_R+I_Y+I_B=0$

Line current (I_L) = Phase Current (I_P)

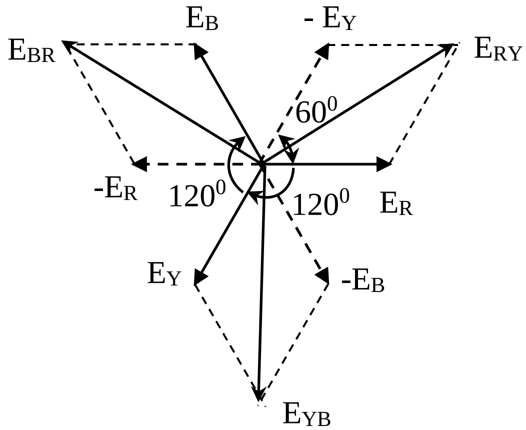


Fig.3b:

Line voltage from above phasor diagram

$$E_{RY} = E_R - E_Y = E_R + (-E_Y)$$

Its magnitude

$$E_{RY} = \sqrt{E_R{}^2 + E_Y{}^2 + 2E_R E_Y \cos 60^0}$$

For balanced 3-phase system

$E_R = E_Y = E_B = E_P$ (Phase Voltage)

So $$E_{RY} = \sqrt{E_P{}^2 + E_P{}^2 + 2E_P E_P \cos 60^0} = \sqrt{E_P{}^2 + E_P{}^2 + 2E_P{}^2 \times \frac{1}{2}}$$

$$E_{RY} = \sqrt{3}E_P$$

Similarly

$$E_{YB} = E_Y + (-E_B) = \sqrt{E_Y{}^2 + E_B{}^2 + 2E_Y E_B \cos 60^0} = \sqrt{3}E_P$$

$$E_{BR} = E_B + (-E_R) = \sqrt{E_B{}^2 + E_R{}^2 + 2E_B E_R \cos 60^0} = \sqrt{3}E_P$$

Hence for balanced system

$E_{RY} = E_{YB} = E_{BR} = \sqrt{3}E_P = E_L$

If ϕ is the angle between phase voltage and phase current then

Active power of 3 phase $=3\ E_P I_P \text{Cos}\ \phi = 3\ \dfrac{E_L}{\sqrt{3}} I_L \text{Cos}\ \phi = \sqrt{3} E_L I_L \text{Cos}\ \phi$ W

Reactive power of 3 phase $=3\ E_P I_P \text{Sin}\ \phi = 3\ \dfrac{E_L}{\sqrt{3}} I_L \text{Sin}\ \phi = \sqrt{3} E_L I_L \text{Sin}\ \phi$ VAR

Apparent power of 3 phase $=3\ E_P I_P = 3\ \dfrac{E_L}{\sqrt{3}} I_L = \sqrt{3} E_L I_L$ VA

Mesh or Delta (Δ) connected 3- ϕ System

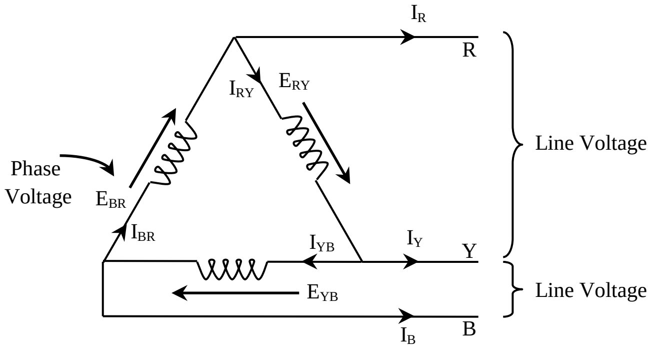


Fig.4a

E_{RY}, E_{YB} & E_{BR} are called phase voltages.

I_{RY}, I_{YB} & I_{BR} are called phase currents.

From figure it is clear that

$E_{RY}+E_{BR}+E_{BY}=0$

Line Voltage (E_L) = Phase Voltage (E_P)

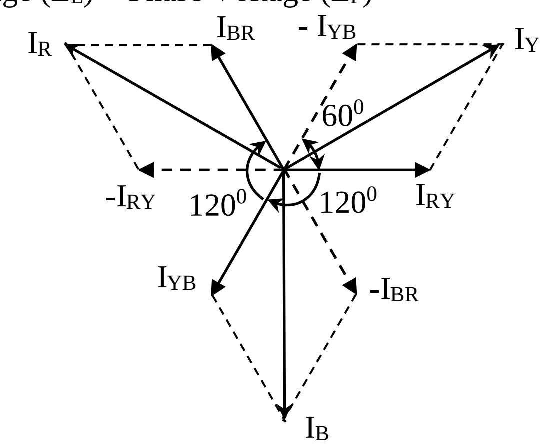


Fig.4b

Apply KCL at node R

$$I_R = I_{BR} - I_{RY} = I_{BR} + (-I_{RY})$$

Its magnitude from above phasor diagram

$$I_R = \sqrt{I_{BR}^2 + I_{RY}^2 + 2I_{BR}I_{RY}\cos 60^0}$$

For balanced 3-phase system

$I_{RY} = I_{YB} = I_{BR} = I_P$ (Phase Current)

So $$I_R = \sqrt{I_P^2 + I_P^2 + 2I_PI_P\cos 60^0} = \sqrt{I_P^2 + I_P^2 + 2I_P^2 \times \frac{1}{2}}$$

$$I_R = \sqrt{3}I_P$$

Similarly

$$I_Y = I_{RY} + (-I_{YB}) = \sqrt{I_p^2 + I_P^2 + 2I_pI_P\cos 60^0} = \sqrt{3}I_P$$

$$I_Y = I_{YB} + (-I_{BR}) = \sqrt{I_p^2 + I_P^2 + 2I_pI_P\cos 60^0} = \sqrt{3}I_P$$

Hence for balanced system

$I_R = I_Y = I_B = I_L = \sqrt{3}I_P$

If ϕ is the angle between phase voltage and phase current then

Active power of 3 phase $= 3\, E_P I_P \text{Cos}\,\phi = 3\, E_L \frac{I_L}{\sqrt{3}} \text{Cos}\,\phi = \sqrt{3} E_L I_L \text{Cos}\,\phi$ W

Reactive power of 3 phase $= 3\, E_P I_P \text{Sin}\,\phi = 3\, E_L \frac{I_L}{\sqrt{3}} \text{Sin}\,\phi = \sqrt{3} E_L I_L \text{Sin}\,\phi$ VAR

Apparent power of 3 phase $= 3\, E_P I_P = 3\, E_L \frac{I_L}{\sqrt{3}} = \sqrt{3} E_L I_L$ VA

Example 1: If the phase voltage of a three phase star connected alternator is E_P. What will be the line voltage?

i. When the phases are correctly connected?
ii. When the connection to one phase is reversed?

Solution:

i. Line voltage (E_L) = Phase Voltage (E_P)

ii. Let connation of phase R is reversed

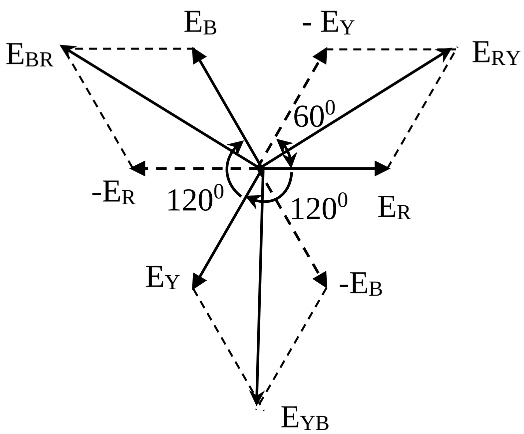


Fig.5a: Phasor Diagram when phases are correctly connected

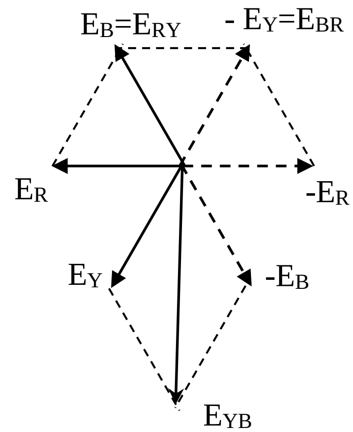


Fig.5b: Phasor Diagram when connection of phase R is reversed

Line voltage from above phasor diagram

$$E_{RY} = E_R - E_Y = E_R + (-E_Y)$$

Its magnitude

$$E_{RY} = \sqrt{E_R{}^2 + E_Y{}^2 + 2E_R E_Y \cos 120^0}$$

$$E_{RY} = \sqrt{E_P{}^2 + E_P{}^2 + 2E_P E_P \cos 120^0} = \sqrt{E_P{}^2 + E_P{}^2 + 2E_P{}^2 \times -\frac{1}{2}}$$

$$E_{RY} = E_P$$

Similarly

$$E_{YB} = E_Y + (-E_B) = \sqrt{E_Y{}^2 + E_B{}^2 + 2E_Y E_B \cos 60^0} = \sqrt{3}E_P$$

$$E_{BR} = E_B + (-E_R) = \sqrt{E_B{}^2 + E_R{}^2 + 2E_B E_R \cos 120^0} = E_P$$

Example 2: Three identical of 20 ohm are connected in star to a 415 V, 3-phase, 50 Hz supply. Calculate

i. Total power taken by load
ii. Power consumed in the resistance if they are connected in delta to same supply.
iii. If one of the resistance is open circuited in each case calculated the power consumed.

Solution:

i. Power in star

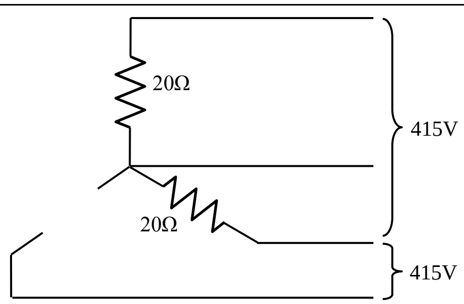
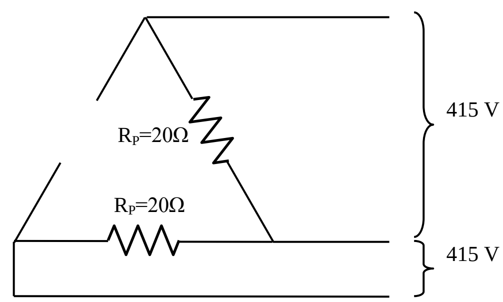
V_L=415 V, $V_L=\sqrt{3}V_P$=415 V ⇒ $V_P=415/\sqrt{3}$ Volts
$I_L=I_P=V_P/R_P=415/20\sqrt{3}$ Amp

Total power consumed (Resistive load⇒ PF=1)
$P=3V_PI_P\cos\phi=3*(415/\sqrt{3})*(415/20\sqrt{3})*1=8611.25$ Watts

ii. Power in Delta

V_L=415 V, $V_L=V_P$=415 V
$I_L=I_P=V_P/R_P=415/20$ Amp
Total power consumed
$P=3V_PI_P\cos\phi=3*(415)*(415/20)*1=25833.75$ Watts

iii. Power when one resistance is open circuit

Star Connected	**Delta Connected**
 Fig.6a $$P=\frac{V^2}{R_{eq}}=\frac{415^2}{40}=4305.625\ Watt$$	 Fig.6a $$P=\frac{V^2}{R_p}+\frac{V^2}{R_p}=\frac{415^2}{20}+\frac{415^2}{20}$$ $$=17222.5\ Watt$$

Measurement of power in 3-phase load: There are three methods

1. One wattmeter method
2. Two wattmeter method
3. Three wattmeter method

1. One wattmeter method :

It is used only for balanced load

Total power consumed by 3-ϕ load

P=3x Power consumed by one phase load
=3x Reading of one wattmeter (W)

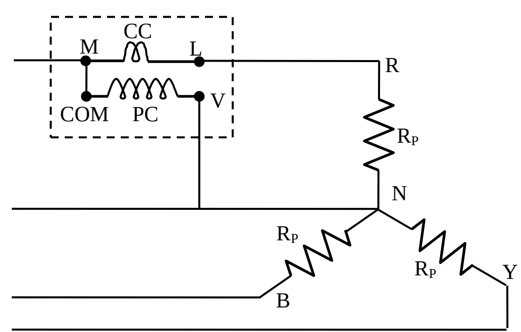


Fig.7a: 3-ϕ Star Connected Balanced Load

Note: if load is delta connected, voltage terminal (V) of PC will be connected to ground.

2. **Three wattmeter method:**

For balanced or unbalanced load total power

$P=W_1+W_2+W_3$

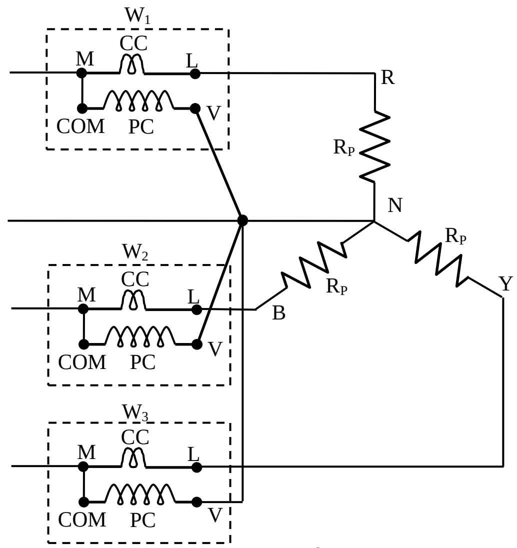


Fig.7b

Note: If load is delta connected, voltage terminal (V) of PC of all wattmeters will be connected to ground.

3. **Two Wattmeter method:**
   a. **When load is star connected:**

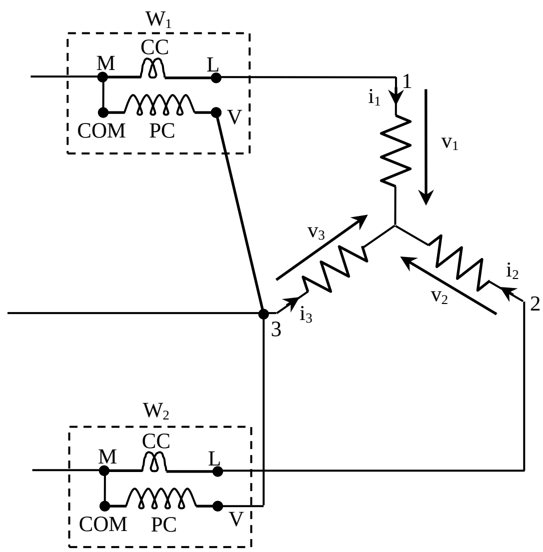


Fig.7 c

Instantaneous power given by wattmeter 1

$p_1 = i_1.(v_1 - v_3)$ ---(1)

Instantaneous power given by wattmeter 2

$p_2 = i_2.(v_2 - v_3)$ ---(2)

Adding (1) & (2)

$p1 + p2 = i_1.(v_1 - v_3) + i_2.(v_2 - v_3)$

$= v_1 i_1 - v_3 i_1 + v_2 i_2 - v_3 i_2$

$= v_1 i_1 + v_2 i_2 - v_3 (i_1 + i_2)$

$= v_1 i_1 + v_2 i_2 + v_3 i_3$ $\left(\because\ i_1 + i_2 + i_3 = 0 \Rightarrow i_1 + i_2 = -i_3\right)$

= Total instantaneous power in 3-phase load

   b. **When load is delta connected:**

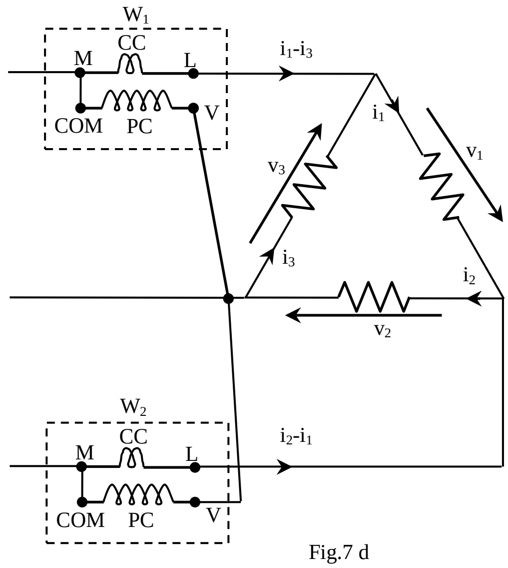


Fig.7 d

Instantaneous power given by wattmeter 1

$p_1=-v_3.(i_1-i_3)$ ---(1)

Instantaneous power given by wattmeter 2

$p_2=v_2.(i_2-i_1)$ ---(2)

Adding (1) & (2)

$p1+p2= -v_3.(i_1-i_3)+ v_2.(i_2-i_1)$

$=-v_3i_1-v_3i_3+v_2i_2-v_2i_1$

$=-(v_2+v_3)i_1+v_2i_2+v_3i_3$

$=v_1i_1+v_2i_2+v_3i_3$ $\left(\because\ v_1+v_2+v_3=0 \Rightarrow v_2+v_3=-v1\right)$

= Total instantaneous power in 3-phase load

Hence a load may be balanced or unbalanced, star connected or delta connected, total instantaneous power of 3-ϕ load will be the sum of the instantaneous power given by the two wattmeters.

Since wattmeter measures active power so total active power of 3-ϕ load will be

$P= W_1+W_2$

W_1= Active power measured by wattmeter 1

W_2= Active power measured by wattmeter 2

Determination of power factor: Consider a star connected balanced load

Let

$V_1= V_2 = V_3=V_p$ Phase voltages (RMS)

$I_1= I_2 = I_3=I_p$ Phase currents (RMS)

Reading of wattmeter 1 $W_1=I_1V_{13}Cos(30-\phi)$

$=I_P\sqrt{3}V_P\ Cos(30-\phi)$

$= \sqrt{3}V_P\ I_P\ Cos(30-\phi)$ ---(1)

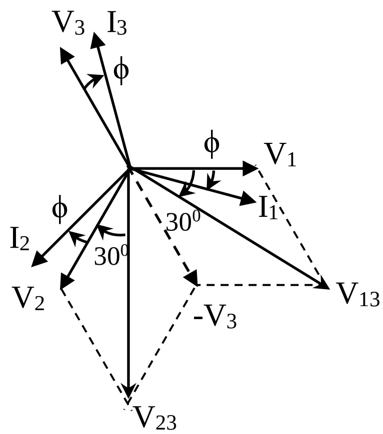


Fig.3b

Reading of wattmeter 2 $W_2 = I_2 V_{23} Cos(30+\phi)$

$= I_P \sqrt{3} V_P\ Cos(30+\phi)$

$= \sqrt{3} V_P\ I_P\ Cos(30+\phi)$ ---(2)

$W_1+W_2 = 3V_P\ I_P\ Cos\phi$=Total active power of 3-ϕload ---(3)

$W_1-W_2 = \sqrt{3} V_P\ I_P\ Sin\phi$ ---(4)

(4)/(3)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\sqrt{3} V_P I_P Sin\phi}{3 V_P I_P Cos\phi}$$

$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \Rightarrow \phi = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

Power factor

$$Cos\phi = Cos\tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

Effect of power factor on wattmeter readings:

We know

$W_1 = V_L I_L Cos(30-\phi)$

$= \sqrt{3} V_P I_P\ Cos\ (30-\phi)$ --- (1)

$W_2 = V_L I_L Cos(30-\phi)$

$= \sqrt{3} V_P I_P\ Cos\ (30+\phi)$ --- (2)

- When $\phi=0$ i.e PF $Cos\phi=1$:

  $W_1 = V_L I_L Cos(30) = V_L I_L(\sqrt{3}/2)$

  $W_2 = V_L I_L Cos(30) = V_L I_L(\sqrt{3}/2)$

  Both wattmeters will show equal readings

- When $\phi=60^0$ i.e PF $Cos\phi=0.5$:

  $W_1 = V_L I_L Cos(30-60) = V_L I_L(\sqrt{3}/2)$

  $W_2 = V_L I_L Cos(30+60) = 0$

  One Wattmeter will show zero reading and total power consumed = W_1

- When ϕ=90 i.e PF Cosϕ=0:

$$W_1 = V_L I_L Cos(30-90) = V_L I_L (1/2)$$
$$W_2 = V_L I_L Cos(30+90) = -V_L I_L (1/2)$$

Hence we can say when power factor angle is greater than 60^0 one wattmeter will give negative reading. For obtaining the reading of that wattmeter either the connection of current coil(CC) or pressure coil(PC) should be changed and reading will be taken as negative.

Example 3: For a certain load one wattmeter reads 20 KW and other 5 KW after the voltage of this wattmeter has been reversed. Calculate power and power factor of the load.
Solution:

Power $P=W_1+W_2=20+(-5)=15$ KW

$$Cos\phi = Cos\tan^{-1}\sqrt{3}\left(\frac{W_1-W_2}{W_1+W_2}\right) = Cos\tan^{-1}\sqrt{3}\left(\frac{20-(-5)}{20+(-5)}\right) = Cos\tan^{-1}\sqrt{3}\left(\frac{25}{15}\right) = Cos70.89^0 = 0.327$$

Q 1. Draw connection diagram fro measurement of power in 3-phase Y connected load using two wattmeter method. In one such experiment the load supplied was 30 KW at 0.7 power factor lagging. Find the reading of each wattmeter.

Hint: W1+W2=P= 30 KW

$$Cos\phi = \tan^{-1}\sqrt{3}\left(\frac{W_1-W_2}{W_1+W_2}\right) = 0.7$$